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Contra - Continuous Functions Between Topological Quotient Spaces

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Abstract:

In this search, we introduce and study new types of contra-continuous functions that is contra-continuous functions between quotient spaces and impact of this by canonical projection functions and composition of this together with some types of contra-continuous functions. Also study image of some types of compact (connected, strongly-s- closed) sets in these quotient spaces under compose of these types of contra-continuous functions.

1. Introduction:

In 1996, Dontchev introduce a new class of continuous functions called contra- continuous functions, in 1999 Jafari and Noiri introduce and studied a new type of contra-continuous functions called contra super-continuous function and, in 2001 they present and study another new type of these functions called α-contra– continuous functions. In 2008, Jawad K. Judy study and introduce some types of contra-continuous functions between topological spaces in his master theses, which presented to college of mathematics and computers sciences at Kufa University. Let (X,τ) be a topological space we recall that: let Q be an equivalent relation on X, let X/Q be the set of all equivalent classes $[x]$ where $[x]=\{y\in X:(y,x)\in Q\}$ of all $x\in X$ and $q:X\longrightarrow X/Q$ be the canonical projection function then quotient topology is the largest topology which makes $q$is a continuous function. We say that $f:(X,\tau)\longrightarrow(Y,S)$ is contra-continuous function if the invers image of all an open set in Y is a closed set in X. Let $T:P(X)\longrightarrow P(X)$ (where $P(X)$ is a power set of the set X) be a function such that $\omega\subseteq T(\omega)\forall\,\omega\in\tau$ then we say that T is an operator topology associated with the topology τ on non-empty set X and the triple (X,τ,T) is an operator topological space and we shill represent to it by symbol (O.T.S.) [1].

2. Definitions and Examples:

2.1 Definitions: [2]

1. Let (X,τ,T) be an operator topological space (O.T.S.) $A\subseteq X$ then we say that: A is T-open (IT- open) if for each $x\in A$ there exist $G\in\tau$ such that $x\in G\subseteq T(G)\subseteq A$ [$x\in G\subseteq int(T(G))\subseteq A$ resp.] where $int(X)$ is interior of the set X According to τ.

2. Let (X, τ, T) be an operator topological space (O.T.S.) then we say that:

X Is T-compact (IT-compact) if for each T-open (IT- open) cover has a finite sub-cover.

Note:

The complement of T-open (IT- open) set is T-closed (IT- closed).

2.2 Definitions: [2]

1. Let (X, τ, T) be an operator topological space (O.T.S.) then we say that:

X Is T-connected (IT- connected) if it is not union of two disjoint non empty T-open (IT- open) subset of X.

2. Let (X, τ, T) be an operator topological space (O.T.S.) then we say that:

X Is T-strongly-S-closed (IT-strongly-S-closed) if every T-closed (IT-closed) cover of X has a finite sub cover.

2.3 Examples:

1. Suppose that (R, τ_u) (real numbers with usual topology) and suppose that Q be an equivalent relation on R such that $Q = \{(r_1, r_2): r_1 - r_2 \in \boldsymbol{Q}\}$ where $\boldsymbol{Q}$ is rational numbers then:

$X/Q = \{[x]: x \in R\} = \{x + p: x \in R, p \in \boldsymbol{Q}\}$.

2. Suppose that: $f: (R, \tau_D) \longrightarrow (R, \tau_u)$ where (R, τ_D) is real numbers with discrete topology, then f is contra-continuous function.

3. Let $X = \{1,2,3,4\}, \tau = \{\{1\}, \{1,2\}, \{1,2,3\}, \{3\}, \{1,3\}, \emptyset, X\}$ then $\tau_{closed} = \{\{2,3,4\}, \{3,4\}, \{1,2,4\}, \{4\}, \{2,4\}, \emptyset, X\}$, and let: $T(A) = CL(A), \forall A \subseteq X$ Where $CL(A)$ indicated to the closure of the set A Then (X, τ, T) is O.T.S. So: $A_1 = \{3\}$ Is open and IT-open.
$A_2 = \{2,3\}$ Is IT-open but not T-open.

4. (R, τ_u) Is IT-connected (T-connected).

5. X=The set of all positive integer numbers and:
$, \tau = \{\{1\}, \{2\}, \{1,2\}, \emptyset, X\}$ Then (X, τ) is IT-strongly-S-closed.

3. Auxiliary Results:

In this part, we set and prove some theorems, which we need in our search:

Note:

We indicate to the quotient topology by symbol τ_q.

We indicate to the function from $P(X/Q)$ to $P(X/Q)$ by symbol T_q (i.e. $T_q: P(X/Q) \longrightarrow P(X/Q)$).

3.1Theorem:

If $q: X \longrightarrow X/Q$ is a canonical projection function and $A \subseteq X$ then:
$q(int(A)) = int(q(A))$.

Proof:

Let $[x] \in q(int(A))$ then $q^{-1}([x]) \in int(A) \subseteq A$

So, $[x] \in q(int(A)) \subseteq q(A)$

Since $int(A)$ is an open set and by definition of canonical projection function and quotient topology we get $q(int(A))$ is an open set

So, by definition of an open set we get $[x] \in int(q(A))$

Now let $[x] \in int(q(A)) \subseteq q(A)$

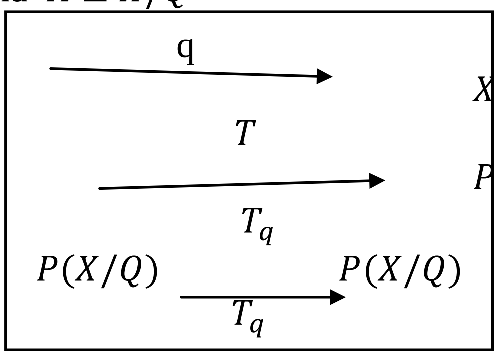
Then $q^{-1}([x]) \in q^{-1}(int(q(A))) \subseteq A$
Since $int(q(A))$ is an open set and q is a continuous function
Then $q^{-1}(int(q(A)))$ is an open set and so, $q^{-1}([x]) \in int(A)$
So, $[x] \in q(int(A))$
Thus $int(q(A)) = q(int(A))$
With the same way, we can prove this theorem for the function q^{-1}●

3.2 Theorem:

Suppose that (X, τ) is a topological space and let $T: P(X) \longrightarrow P(X)$ such that$q^{-1}(T_q(\acute{\omega})) = T(q^{-1}(\acute{\omega})), \forall \acute{\omega} \subseteq X/Q$, then if (X, τ, T) is O.T.S. Then $(X/Q, \tau_q, T_q)$ is also O.T.S.

Proof:

Since (X, τ, T) is O.T.S. and $T: P(X) \longrightarrow P(X)$, Then $\omega \subseteq T(\omega), \forall \omega \in \tau$.
For quotient topology $(X/Q, \tau_q)$, $T_q: P(X/Q) \longrightarrow P(X/Q)$
The canonical projection function $q: X \longrightarrow X/Q$ is continuous function. Suppose that $\acute{\omega} \subseteq X/Q$ such that $\acute{\omega} \in \tau_q$(i.e. $\acute{\omega}$ is an open set in X/Q)
Then $q^{-1}(\acute{\omega}) \subseteq q^{-1}(X/Q)$ and $q^{-1}(\acute{\omega}) \subseteq X$ which an open set.
Since $q^{-1}(\acute{\omega}) \subseteq X$ which an open set and (X, τ, T) is O.T.S.
Then $q^{-1}(\acute{\omega}) \subseteq T(q^{-1}(\acute{\omega}))$
So, $q(q^{-1}(\acute{\omega})) \subseteq q(T(q^{-1}(\acute{\omega})))$ and since q is surjection.
We get $\acute{\omega} \subseteq q(T(q^{-1}(\acute{\omega})))$
Now, since $q^{-1}(\acute{\omega}) \subseteq X$ then $q^{-1}(\acute{\omega}) \in P(X)$ So, $T(q^{-1}(\acute{\omega})) \in P(X)$
And since if $A \in P(X)$ then $A \subseteq X$ So $q(T(q^{-1}(\acute{\omega}))) \subseteq X/Q$
And then $q(T(q^{-1}(\acute{\omega}))) \in P(X/Q)$
Now to prove that $T_q(\acute{\omega}) = q(T(q^{-1}(\acute{\omega})))$
Let $A = T_q(\acute{\omega})$
Since $T_q$is an open map, then $T_q(\acute{\omega})$ is an open set and $A \subseteq X/Q$
Since q is surjection then $q^{-1}(A) \subseteq X$
Since (X, τ, T) is O.T.S. then $q^{-1}(A) \subseteq T(q^{-1}(A))$
Since $T(q^{-1}(A)) \in P(X)$ So $T(q^{-1}(A)) \subseteq X$
So, $T_q(\acute{\omega}) \subseteq q(T(q^{-1}(T_q(\acute{\omega}))))$
Now, by using the condition:
$q^{-1}(T_q(\acute{\omega})) = T(q^{-1}(\acute{\omega}))$
We get: $T_q(\acute{\omega}) = q(T(q^{-1}(\acute{\omega})))$
And the proof is complete●

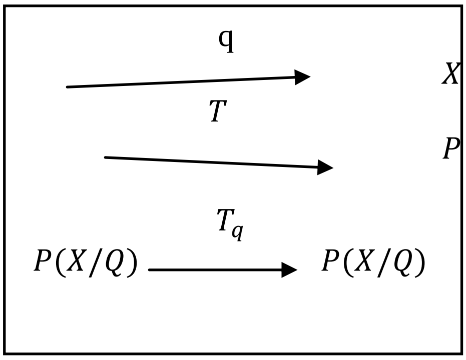


The following diagram explain relation between above functions
Now, by using theorem 3.2 we can prove the following theorems:

3.3 Theorem:

Suppose that (X, τ, T) is O.T.S. And $A \subseteq X$ such that A is T-open (IT-open) in X, then $q(A)$ is T-open (IT-open) in X/Q where Q is equivalent relation.
Proof:

For a continuous function $q: X \longrightarrow X/Q$
Suppose that $[x] \in q(A) \subseteq X/Q$
Since q is surjection map
Then for each $[x] \in q(A), \ni x \in X$ such that $x = q^{-1}([x])$
Since $A \subseteq X$ is T-open (IT-open) such that $T: P(X) \longrightarrow P(X)$
Then for each $x \in A \ \ni G \in \tau$ such that $x \in G \subseteq T(G) \subseteq A$ $[int(T(G)) \subseteq A\ resp.]$
Since $x = q^{-1}([x]) \in A$
So, there exist $G \in \tau$ such that $q^{-1}([x]) \in G \subseteq T(G) \subseteq A$ $[int(T(G)) \subseteq A\ resp.]$
Then, $q(q^{-1}([x])) \in q(G) \subseteq q(T(G)) \subseteq q(A)$ $[q(int(T(G)) \subseteq q(A)] resp.$).
By use theorem 3.1, we get:
$[x] \in q(G) \subseteq q(T(G)) \subseteq q(A)$ $[int(q(T(G)) \subseteq q(A)]\ resp.$)
Now, by using definition of canonical projection function $q: X \longrightarrow X/Q$
And quotient topology τ_q we get $q(G)$ is an open set.
Since $G \subseteq X$ then $G \in P(X)$ and $T(G) \in P(X)$
So, $T(G) \subseteq X$ then $q(T(G)) \subseteq X/Q \in P(X/Q)$
So, $q(T(G)) \in P(X/Q)$
Also, for $T_q(q(G))$ since $G \subseteq X$ then $q(G) \subseteq X/Q \in P(X/Q)$
Thus $T_q(q(G)) \subseteq P(X/Q)$
Now, to prove $T_q(q(G)) = q(T(G))$
By difinition of canonical projection function and quotint topology there exist an open set G such that:
$\acute{\omega} = q(G)$ which an open in X/Q
From theorem 3.2, we get:
$q^{-1}(T_q(\acute{\omega})) = T(q^{-1}(\acute{\omega}))$
So, $q^{-1}(T_q(q(G))) = T(q^{-1}(q(G)))$
Thus $q^{-1}(T_q(q(G))) = T(G)$
So, $T_q(q(G)) = q(T(G))$
Then $q(A)$ is T-open (IT-open) ●



3.4 Theorem:

If $A \subseteq X$ such that A is T-strongly-S-closed (IT-strongly-S-closed) then $q(A) \subseteq X/Q$ is T-strongly-S-closed (IT-strongly-S-closed).

Proof:

Let $\{B_i\}_{i \in I}$ be a T-closed (IT-closed) cover of $q(A)$
Then $q(A) \subseteq \bigcup_{i \in I} B_i$
So, $q^{-1}(q(A)) \subseteq q^{-1}(\bigcup_{i \in I} B_i)$
Then $A \subseteq q^{-1}(\bigcup_{i \in I} B_i) = \bigcup_{i \in I} q^{-1}(B_i)$
Since $q: X \longrightarrow X/Q$ is a continuous function
We get, $\bigcup_{i \in I} q^{-1}(B_i)$ is a T-closed (IT-closed) set
Since A is T-strongly-S-closed (IT-strongly-S-closed)
Then there exist a finite set $\boldsymbol{I}$ of T-close (IT-close) cover of A
This mean $A \subseteq \bigcup_{i \in \boldsymbol{I}} q^{-1}(B_i)$
So, $q(A) \subseteq q(\bigcup_{i \in \boldsymbol{I}} q^{-1}(B_i)) = \bigcup_{i \in \boldsymbol{I}} q(q^{-1}(B_i)) = \bigcup_{i \in \boldsymbol{I}} B_i$

Thus $q(A)$ is T-strongly-S-closed (IT-strongly-S-closed) ●

3.5 Theorem:

If $q: X \longrightarrow X/Q$ is canonical projection function and $B \subseteq X/Q$ is T-open (IT-open) then $q^{-1}(B)$ is T-open (IT-open).

Proof:

Let $x \in q^{-1}(B)$, then $q(x) \in B$

Since B is T-open (IT-open)

Then there exist $G \in \tau_q$ such that $q(x) \in G \subseteq T_q(G) \subseteq B$ $[int\left(T_q(G)\right) \subseteq B\ resp.]$

So,$q^{-1}(q(x)) \in q^{-1}(G) \subseteq q^{-1}(T_q(G)) \subseteq q^{-1}(B)$ $[q^{-1}(int\left(T_q(G)\right)) \subseteq q^{-1}(B))\ resp.]$

By use theorem 3.1, we get:

$$x \in q^{-1}(G) \subseteq q^{-1}(T_q(G)) \subseteq q^{-1}(B)\ [int(q^{-1}\left(T_q(G)\right) \subseteq q^{-1}(B))\ resp.]$$

Since q is continuous function then $q^{-1}(G)$ is an open set

From theorem 3.2 we get:

$T(q^{-1}(G)) = q^{-1}(T_q(G))$

And so, $x \in q^{-1}(G) \subseteq T(q^{-1}(G)) \subseteq q^{-1}(B)(int\left(T(q^{-1}(G)\right) \subseteq q^{-1}(B)\ resp.)$

Then $q^{-1}(B)$ is T-open (IT-open) ●

4.Main Results:

In this part we study image of a compact (connected, strongly-S-closed) sets under effect of contra-continuous function. Since invers image of an open set under effect of contra-continuous function is a closed set, then image of a compact (connected, strongly-S- closed) set is not compact(connected, strongly-S-closed) under effect of this kind of functions, so we use a composition of these contra-continuous functions with canonical projection function and with the others to ensure access a compact (connected, strongly-S- closed) set. And this is a good result and solution to this problem.

4.1 Theorem:

Suppose that $f: (X, \tau) \longrightarrow (Y, S)$ and $g: (X/Q_1, \tau_{q_1}) \longrightarrow (Y/Q_2, S_{q_2})$(where Q_1, $Q_2$are equivalent relations over X and Y resp.),

Such that $q_2 f = g q_1$ then the following statements are equivalent:

f Is contra continuous function. g Is contra continuous function.

Proof:

$\longrightarrow$ (2)

Let A be an open set in Y/Q_2

Since $q_2: Y \longrightarrow Y/Q_2$ is continuous function

Then $q_2^{-1}(A)$ is an open set in Y

Since f is contra-continuous function

Then $f^{-1}(q_2^{-1}(A))$ is a closed set in X

By definition of canonical projection function q_1 and quotient topology τ_{q_1} we get :

$q_1(f^{-1}(q_2^{-1}(A)))$ is a closed set in X/Q_1
Since $q_2 f = g q_1$ then $g = q_2 f q_1^{-1}$
So, $g^{-1} = q_1 f^{-1} q_2^{-1}$
And the proof is complete.

(2)⟶ (1)

Let B be an open set in Y
So, by definition of canonical projection function q_2 and quotient topology τ_{q_2} we get:

$$\begin{tikzcd} X \arrow[r, "q_1"] \arrow[d, "f"'] & X/Q_1 \arrow[d, "g"] \\ Y \arrow[r, "q_2"'] & Y/Q_2 \end{tikzcd}$$

$q_2(B)$ is an open set in Y/Q_2
Since g is contra-continuous function
Then $g^{-1}(q_2(B))$ is a closed set in X/Q_1
Since q_1 is continuous function
Then $(q_1^{-1}(g^{-1}(q_2(B)))$ is a closed set in X
Since $q_2 f = g q_1$ then $f = q_2^{-1} g q_1$ and $f^{-1} = q_1^{-1} g^{-1} q_2$
And the proof is complete●

4.2 Theorem:

Suppose that $f:(X/Q, \tau_q) \longrightarrow (X/Q, \tau_q)$ is contra-continuous function such that $q: X \longrightarrow X/Q$ and $q = fq$, let $A \subseteq X/Q$is a compact set then $f(A)$ is a compact.

Proof:

Let $\{B_i\}_{i\in I}$ be an infinite open cover of $f(A)$ in X/Q
Then $f(A) \subseteq \bigcup_{i\in I} B_i$
Since q is continuous and surjection
Then $q^{-1}(f(A)) \subseteq q^{-1}(\bigcup_{i\in I} B_i) = \bigcup_{i\in I} q^{-1}(B_i)$ and $\bigcup_{i\in I} q^{-1}(B_i)$ is an open set in X, So, by definition of quotient space and canonical projection function we get
$q(q^{-1}(\bigcup_{i\in I} B_i)) = \bigcup_{i\in I} q(q^{-1}(B_i)) = \bigcup_{i\in I} B_i$ is an open set in X/Q
So, $q(q^{-1}(f(A))) \subseteq q(q^{-1}(\bigcup_{i\in I} B_i)) = \bigcup_{i\in I} q(q^{-1}(B_i)) = \bigcup_{i\in I} B_i$
On the other hand, by using the above condition $q = fq$ we get:

$$\begin{tikzcd} X \arrow[r, "q"] \arrow[dr, "q"'] & X/Q \arrow[d, "f"] \\ & X/Q \end{tikzcd}$$

Then $q(q^{-1} f^{-1}(f(A)) \subseteq \bigcup_{i\in I} q(q^{-1} f^{-1}(B_i)) = \bigcup_{i\in I} B_i$ in X/Q
Thus, $A \subseteq \bigcup_{i\in I} f^{-1}(B_i)$ which is open in X/Q
Since A is a compact set
Then there exist a finite set $\boldsymbol{I}$ such that:
$A \subseteq \bigcup_{i\in \boldsymbol{I}} f^{-1}(B_i)$
So, $f(A) \subseteq f(\bigcup_{i\in \boldsymbol{I}} f^{-1}(B_i)) = \bigcup_{i\in \boldsymbol{I}} f(f^{-1}(B_i)) = \bigcup_{i\in \boldsymbol{I}} B_i$
Thus $f(A)$ is a compact set●

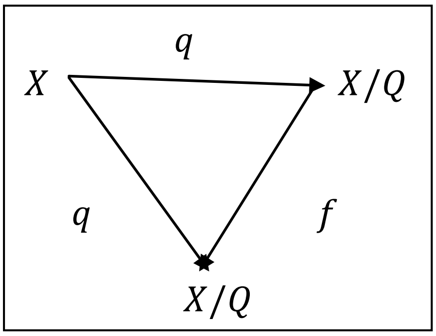
4.3 Theorem:

Suppose that $f:(X/Q, \tau_q) \longrightarrow (X/Q, \tau_q)$ is contra-continuous function such that $q: X \longrightarrow X/Q$ and $q = fq$, let $A \subseteq X$ be T-compact (IT-compact) set then $f(A)$ is a T-compact (IT-compact).

Proof:

Let $\{B_i\}_{i\in I}$ be an infinite T-open (IT-open) cover of $f(A)$ in X/Q

Then $f(A) \subseteq \bigcup_{i\in I} B_i$
Since q is continuous and surjection and theorems 3.2, 3.5 we get:
$q^{-1}(f(A)) \subseteq q^{-1}(\bigcup_{i\in I} B_i) = \bigcup_{i\in I} q^{-1}(B_i)$ And $\bigcup_{i\in I} q^{-1}(B_i)$ is an T-open (IT-open) set in X
So, by definition of quotient space and canonical projection function and by using theorem 3.3 we get:
$q(q^{-1}(\bigcup_{i\in I} B_i)) = \bigcup_{i\in I} q(q^{-1}(B_i)) = \bigcup_{i\in I} B_i$ is T-open (IT-open) set in X/Q
So, $q(q^{-1}(f(A))) \subseteq q(q^{-1}(\bigcup_{i\in I} B_i)) = \bigcup_{i\in I} q(q^{-1}(B_i)) = \bigcup_{i\in I} B_i$
On the other hand, by using the above condition $q = fq$ we get:
Then $q(q^{-1}f^{-1}(f(A))) \subseteq \bigcup_{i\in I} q(q^{-1}f^{-1}(B_i)) = \bigcup_{i\in I} B_i$ in X/Q
Thus, $A \subseteq \bigcup_{i\in I} f^{-1}(B_i)$ which is open in X/Q
Since A is a compact set
Then there exist a finite set $\boldsymbol{I}$ such that:
$A \subseteq \bigcup_{i\in \boldsymbol{I}} f^{-1}(B_i)$
So, $f(A) \subseteq f(\bigcup_{i\in \boldsymbol{I}} f^{-1}(B_i)) = \bigcup_{i\in \boldsymbol{I}} f(f^{-1}(B_i)) = \bigcup_{i\in \boldsymbol{I}} B_i$
Thus $f(A)$ is a T-compact (IT-compact) ●

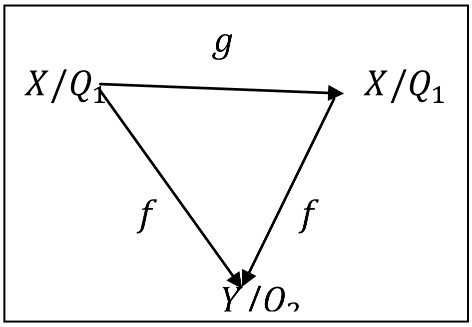


4.4 Theorem:

Suppose that $g: (X/Q_1, \tau_q) \longrightarrow (X/Q_1, \tau_q)$ and $f: (X/Q_2, \tau_{q_1}) \longrightarrow (Y/Q_2, \tau_{q_2})$ are contra-continuous functions such that and $f = fg$, let $A \subseteq X$ be T-compact (IT-compact) set then $f(A)$ is a T-compact (IT-compact).

Proof:

Let $\{B_i\}_{i\in I}$ be an infinite T-open (IT-open) cover of $f(A)$ in Y/Q_2
Then $f(A) \subseteq \bigcup_{i\in I} B_i$
Since f is contra-continuous function and by using theorem 3.5 for B_i we get:
$f^{-1}(f(A)) \subseteq f^{-1}(\bigcup_{i\in I} B_i) = \bigcup_{i\in I} f^{-1}(B_i)$ And $\bigcup_{i\in I} f^{-1}(B_i)$ is an T-closed (IT-closed) set in X/Q_1
So, $A \subseteq \bigcup_{i\in I} f^{-1}(B_i)$ is a closed set in X/Q_1
Since g is contra-continuous function
Then $g^{-1}(A) \subseteq g^{-1}(f^{-1}(\bigcup_{i\in I} B_i))$ is an open set in X/Q_1, So, $g^{-1}(A) \subseteq \bigcup_{i\in I} g^{-1}(f^{-1}(B_i))$
Now, by using the condition $f = fg$
we get $A \subseteq f^{-1}(\bigcup_{i\in I} B_i) = \bigcup_{i\in I} f^{-1}(B_i)$
$A \subseteq \bigcup_{i\in I} f^{-1}(B_i)$ is T-open (IT-open) set in X/Q_1
Since A is T-compact (IT-compact) set in X/Q_1
Then there exist a finite set $\boldsymbol{I}$ such that:
$A \subseteq \bigcup_{i\in I} f^{-1}(B_i)$, So, $f(A) \subseteq f(\bigcup_{i\in \boldsymbol{I}} f^{-1}(B_i)) = \bigcup_{i\in \boldsymbol{I}} f(f^{-1}(B_i)) = \bigcup_{i\in \boldsymbol{I}} B_i$
Thus $f(A)$ is a T-compact (IT-compact) ●



With same way of proof theorem 4.4, we can prove the following theorems:

4.5 Theorem:

Suppose that $g: (X/Q_1, \tau_{q_1}) \longrightarrow (X/Q_1, \tau_{q_1})$ and $f: (X/Q_1, \tau_{q_1}) \longrightarrow (Y/Q_2, \tau_{q_2})$are contra-continuous functions such that and $f = fg$, let $A \subseteq X/Q_1 X$ be T-connected (IT-connected) set then$f(A)$ is a T-connected (IT- connected).

4.6 Theorem:

Suppose that $g:(X/Q_1,\tau_{q_1}) \longrightarrow (X/Q_1,\tau_{q_1})$ and $f:(X/Q_1,\tau_{q_1}) \longrightarrow (Y/Q_2,\tau_{q_2})$are contra-continuous functions such that and $f = fg$, let $A \subseteq X/Q_1X$ be T-strongly-S-closed (IT-strongly-S-closed) set then $f(A)$ is a T-strongly-S-closed (IT-strongly-S-closed).

REFERENCES:

[1] Mustafa, Hadi J. and Hassan, Ali Abdul, (T-open sets), MSC. Thesis, Muuta University, Jordan (2004).
[2] Mustafa, Hadi J. and Kadima, H.H., (On T* -connected and T* compact).
[3] Judi, Jawad K. (Certain Types of Contra-Continuous Functions),Master Degree Thesis College of Mathematics and Computer Sciences\Kufa University 2009.
[4] Dontchev, J., (1996), (Contra- Continuous functions and Strongly S– Closed Spaces) , Internal J. Math.Sci.19(2)(1996),303-310.
[5] Jesper M. Moller,(General Topology), 2nd April 2005.
[6] د. غفار حسين موسى (مقدمة في التبولوجيا) ، كلية العلوم جامعة الزرقاء ، دار المسيرة للنشر والتوزيع والطباعة، عمان –الطبعة الاولى 2002.

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الخلاصة:

في هذا البحث درسنا أنماط جديدة من الدوال المستمرة المضادة بين فضاءات القسمة التبولوجية ولنتذكر معا" تعريف الدالة المستمرة بصورة مضادة، يقال للدالة $f:(X,\tau) \longrightarrow (Y,S)$ انها مستمرة بصورة مضادة اذا كانت الصورة العكسية لكل مجموعة مفتوحة في Y تبعا" ل S مجموعة مغلقة في X تبعا" ل τ . دراستنا في هذا البحث كانت بالاعتماد على دالة الاسقاط القانوني q (المُعرفة لتبولوجي القسمة على الفضاء التبولوجي قيد الدراسة) وذلك بتركيبها مع بعض الدوال المستمرة المضادة، حيث كان الهدف الرئيسي من النتائج الثانوية البرهنة على انه اذا كان الفضاء التبولوجي (X,τ) فضاء تبولوجي مؤثر (O.T.S.) فأن فضاء القسمة التبولوجي $(X/Q,\tau_q)$ هو أيضا" (O.T.S.). كذلك درسنا في النتائج الرئيسية صور المجموعات المرصوصة والمغلقة بقوة والمتصلة في فضاءات القسمة السابقة الذكر تحت تأثير تركيب الدوال المذكورة انفا" (دالة الاسقاط القانوني مع دالة مستمرة بصورة مضادة أو دوال مستمرة مضادة مع بعضها البعض) وكما هو معلوم فأن صورة المجموعة المرصوصة تحت تأثير الدالة المستمرة بصورة مضادة لا يمكن ان يكون مجموعة مرصوصة كذلك الحال بالنسبة لصورة المجموعة المغلقة بقوة والمتصلة تحت تأثير الدالة المستمرة بصورة مضادة، لكننا باستخدام طريقة التركيب للدوال السابقة الذكر استطعنا البرهنة على انها مجموعة مرصوصة (مغلقة بقوة، متصلة).