On (Semi, Pre, Semi-pre, b)-open subgraph

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Abstract: Due it difficult to find applications in topological spaces, which are branches of pure mathematics. The importance of this paper is to find applications in graph theory. So, We Introduced (semi, pre, b, semi-pre)-open subgraph to graph theory. The relations among them were studied. At least many theorems were proofed as a characterization and some examples introduced to explain the subject.

Keywords: *Graph theory, Subgraph, Topology, Closure, Interior*.

1. Introduction

A graph theory [1] is one of the primary topics in modern mathematics. This theory is also used in most branches of knowledge. It serves us as a simplified mathematical model for any system that involves dual operation. In this paper, we studied a simple graph. Topology extends to the era of Greek civilization, where the Greeks studied the concept of continuity [2], but topology did not appear in its current state until the beginning of this century when Frechet published in 1906 his thesis that dealt with the conjunction between the distance between him and the concept of continuity, but the worlds Riesz and Hausdorff between us and not it is necessary for this coupling, it is possible to study continuity without indicating the coupling of distance, and therefore the so-called topology [3]. Modern study has been studied the connection between graph theory and topology. Mohamed Atef [4] studied in his thesis adjacent between vertices and obtained many results. In this paper, studied is not adjacent between vertices. Also, some researchers make the relation on vertices of a graph only and others made it on edges of constructing topological space for any graph [5, 6, 7, 8]. But the process is to establish a graph from a given topology is not interested before.

2. Preliminaries

Definition 2.1[1, 9]

"A graph G is a triple consisting of a vertex set $\grave{\grave{V}}(G)$, an edge set $\grave{\grave{E}}(G)$, and a relation that associates with each edge two vertices (not necessarily distinct) called it`s end point" and we express a graph to arranged pairs ($\grave{\grave{V}}$, $\grave{\grave{E}}$) or ($\grave{\grave{V}}(G)$, $\grave{\grave{E}}(G)$).

Definition 2.2[1, 9]

"Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph, we call H is a subgraph from G if $\grave{\grave{V}}(H) \subseteq \grave{\grave{V}}(G)$, $\grave{\grave{E}}(H) \subseteq \grave{\grave{E}}(G)$, in this case we would write $H \subseteq G$. The spanning subgraph from a graph G is a subgraph acquired by edge deletions only. A deduced subgraph of a graph G is a subgraph acquired by vertex deletions along with the incident edges".

Definition 2.3 [2, 3]

"If Y is non-empty set, a collection" $\tau \subseteq P(Y)$ is called topology on Y if the following holds:

(1) $Y, \emptyset \in \tau$.

(2) The intersection of a finite number of elements in τ , $is\ in\ \tau$.

(3) The union of a finite or infinite number of elements of sets in $\tau\ belong\ to\ \tau$.

Then (Y, τ) is called a topological space. Any element in (Y, τ) is called
open set and it is complement is called closed set".

Definition 2.4 [2, 3]

"Let Y is a non-empty set and let τ is the collection of every subsets from Y. Then τ is named the discrete topology on the set Y. The topological space (Y, τ) is called a discrete space. If $\tau = \{Y, \emptyset\}$. Then τ is named indiscrete topology and the topological space (Y, τ) is named an indiscrete topological space".

Definition 2.5 [2, 3]

"Let (Y, τ) be a topological space, $A \subseteq Y$. The closure of A symbolized by $Cl(A)$ is defined as the smallest closed set that includes A.It is thus the intersection of every closed sets that include A".

Definition 2.6 [2, 3]

"Let (Y, τ) be a topological space, $A \subseteq Y$. The interior of A symbolized by $Int(A)$ is defined as the largest open set included in A. It is thus the union of every open sets included in A".

Definition 2.7

A subset A of a space X is called:

(1) "Semi-open [10] if $A \subseteq Cl(Int(A))$".

(2) "Preopen [11] if $A \subseteq Int(Cl(A))$".

(3) "b-open [12] if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$".

(4) "Semi-preopen[13] if $A \subseteq Cl(Int(Cl(A)))$".

Definition 2.8 [14, 15]

"Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph, $\grave{\grave{v}} \in \grave{\grave{V}}(G)$ then we define the post stage $\grave{\grave{v}}R$ is the set of all vertices which is not neighborhood of $\grave{\grave{v}}$. S_G is the collection of $(\grave{\grave{v}}R)$ is called subbasis of graph". $\beta_G = \bigcap_{i=1}^{n} S_{G_i}$ is called bases of graph. Then the union of β_G is form a topology on G and $(\grave{\grave{V}}(G), \tau_G)$ is called topological graph.

Example 2.9

Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph (see figure 1). We construct a topological space on G as follows:

$\grave{\grave{v}}_1R = \{\grave{\grave{v}}_4\}, \grave{\grave{v}}_2R = \{\grave{\grave{v}}_3\}, \grave{\grave{v}}_3R = \{\grave{\grave{v}}_2\},$
$\grave{\grave{v}}_4R = \{\grave{\grave{v}}_1\}.$

Then a subbase of a topology is
$S_G = \{\{\grave{\grave{v}}_1\}, \{\grave{\grave{v}}_2\}, \{\grave{\grave{v}}_3\}, \{\grave{\grave{v}}_4\}\}.$

The base is $\beta_G = \{\grave{\grave{V}}(G), \emptyset, \{\grave{\grave{v}}_1\}, \{\grave{\grave{v}}_2\}, \{v_3\}, \{v_4\}\}.$

Therefore, the topological graph on G well be
$\tau_G = \{\grave{\grave{V}}(G), \emptyset, \{\grave{\grave{v}}_1\}, \{\grave{\grave{v}}_2\}, \{\grave{\grave{v}}_3\}, \{\grave{\grave{v}}_4\}, \{\grave{\grave{v}}_1, \grave{\grave{v}}_2\}, \{\grave{\grave{v}}_1, \grave{\grave{v}}_3\}, \{\grave{\grave{v}}_1, \grave{\grave{v}}_4\}, \{\grave{\grave{v}}_2, \grave{\grave{v}}_3\}, \{\grave{\grave{v}}_2, \grave{\grave{v}}_4\}, \{\grave{\grave{v}}_3, \grave{\grave{v}}_4\}, \{\grave{\grave{v}}_1, \grave{\grave{v}}_2, \grave{\grave{v}}_3\}, \{\grave{\grave{v}}_1, \grave{\grave{v}}_3, \grave{\grave{v}}_4\}, \{\grave{\grave{v}}_2, \grave{\grave{v}}_3, \grave{\grave{v}}_4\}\}.$

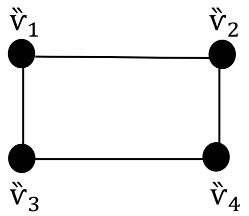

Figure 1. Simple Graph G.

Remark 2.10 [16]

"The complete graph is an indiscrete topology".

Definition 2.11 [16]

"Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph, H be a subgraph from G. Then the graph closure of $\grave{\grave{V}}(H)$ has the shape: $Cl_G(\grave{\grave{V}}(H)) = \grave{\grave{V}}(H) \cup \{\grave{\grave{v}} \in \grave{\grave{V}}(G) : \grave{\grave{v}}R \cap \grave{\grave{V}}(H) \neq \emptyset\}$."

Definition 2.12 [16]

"Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph, H be a subgraph from G. Then the graph interior of $\grave{\grave{V}}(H)$ has the shape: $Int_G(\grave{\grave{V}}(H)) = \{\grave{\grave{v}} \in \grave{\grave{V}}(G) : \grave{\grave{v}}R \subseteq \grave{\grave{V}}(H)\}$".

Example 2.13

From example 2.9. If H, W are subgraphs from G with vertices $\grave{\grave{V}}(H) = \{\grave{\grave{v}}_1, \grave{\grave{v}}_3\}$. Then $\grave{\grave{v}}_2R = \{\grave{\grave{v}}_3\}$, $\grave{\grave{v}}_4R = \{\grave{\grave{v}}_1\}$. So $\grave{\grave{v}}_2R \cap V(H) \neq \emptyset, \grave{\grave{v}}_4R \cap \grave{\grave{V}}(H) \neq \emptyset$. Then $Cl_G(\grave{\grave{V}}(H)) = \grave{\grave{V}}(H) \cup \{\grave{\grave{v}}_2, \grave{\grave{v}}_4\} = \grave{\grave{V}}(G)$, $Int_G(\grave{\grave{V}}(H)) = \{\grave{\grave{v}}_2, \grave{\grave{v}}_4\}$.

3. Definitions On (Semi, Pre, Semi-pre, b)-open subgraph

Definition 3.1

Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph that contains a topological graph $(\grave{\grave{V}}(G), \tau_G)$, H be a subgraph of G is called open subgraph if $Int_G(\grave{\grave{V}}(H)) = \grave{\grave{V}}(H)$. It is called closed subgraph if its complement is open subgraph.

Example 3.2:

Let $G(V, E)$ be a graph (see figure 2), H be a subgraph from G with vertices $\grave{\grave{V}}(H) = \{v_1, v_2, v_3, v_4\}$. Then $\grave{\grave{v}}_1R = \{\grave{\grave{v}}_2, \grave{\grave{v}}_3, \grave{\grave{v}}_4\}, \grave{\grave{v}}_2R = \{\grave{\grave{v}}_1, \grave{\grave{v}}_3, \grave{\grave{v}}_4\}, \grave{\grave{v}}_3R = \{\grave{\grave{v}}_1, \grave{\grave{v}}_2\}$,
$\grave{\grave{v}}_4R = \{\grave{\grave{v}}_1, \grave{\grave{v}}_2\}, \grave{\grave{v}}_5R = \{\grave{\grave{v}}_6, \grave{\grave{v}}_7\}, \grave{\grave{v}}_6R = \{\grave{\grave{v}}_5, \grave{\grave{v}}_7\}, \grave{\grave{v}}_7R = \{\grave{\grave{v}}_5, \grave{\grave{v}}_6\}$,
$Int_G(\grave{\grave{V}}(H)) = \{\grave{\grave{v}}_1, \grave{\grave{v}}_2, \grave{\grave{v}}_3, \grave{\grave{v}}_4\}$.So $\grave{\grave{V}}(H)$ is open subgraph and its complement closed subgraph.

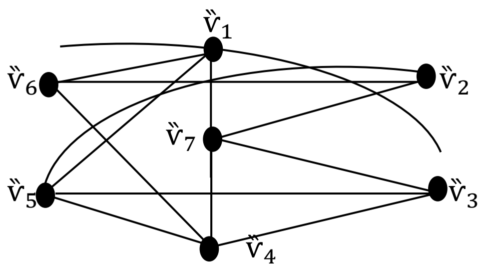

Figure 2. A simple Graph G.

Definition 3.3

Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph that contains a topological graph $(\grave{\grave{V}}(G), \tau_G)$, H be a subgraph of G is called semi-open subgraph if

$\ddot{V}(H) \subseteq Cl_G(Int_G(\ddot{V}(H)))$. The family of all semi-open subgraph from G will be denoted by $SO(\ddot{V}(G))$. The complement of a semi-open subgraph is called a semi-closed subgraph and the family of all semi- closed subgraph from G will be denoted by $SF(\ddot{V}(G))$.

Remark 3.4

Every open subgraph is semi open subgraph.

Definition 3.5

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$", H be a subgraph from $G$is called preopen subgraph if $\ddot{V}(H) \subseteq Int_G(Cl_G(\ddot{V}(H)))$. The family of all preopen subgraph from G will be denoted by $PO(\ddot{V}(G))$. The complement of a preopen subgraph is called pre-closed subgraph and the family of all pre-closed subgraph from G will be denoted by $PF(\ddot{V}(G))$.

Definition 3.6:

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$", H be a subgraph from $G$is called semi-preopen subgraph if $\ddot{V}(H) \subseteq Cl_G(Int_G(Cl_G(\ddot{V}(H))))$. The family of all semi-preopen subgraph from G will be denoted by $SPO(\ddot{V}(G))$. The complement of a semi-preopen subgraph is called a semi-preclosed subgraph and the family of all semi-preclosed subgraph from G will be denoted by $SPF(\ddot{V}(G))$.

Definition 3.7

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$", H be a subgraph from $G$is called b-open subgraph if $\ddot{V}(H) \subseteq Int_G(Cl_G(\ddot{V}(H))) \cup Cl_G(Int_G(\ddot{V}(H)))$. The family of all b-open subgraph of G will be denoted by $bO(\ddot{V}(G))$. The complement of b-open subgraph is called b-closed subgraph and the family of all b-closed subgraph from G will be denoted by $bF(\ddot{V}(G))$.

Definition 3.8: "Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$". Then the semi-closure subgraph is $sCl_G(\ddot{V}(H)) = \cap \{\ddot{V}(F);\ \ddot{V}(F) \text{ is semi} - \text{closed subgraph}, \ddot{V}(H) \subseteq \ddot{V}(F)\}$, and the semi-interior subgraph is $sInt_G(\ddot{V}(H)) = \cup \{\ddot{V}(U);\ \ddot{V}(U) \text{ is semi} - \text{open subgraph}, \ddot{V}(H) \subseteq \ddot{V}(U)\}$.

Definition 3.9

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$". Then the pre-closure subgraph is $pCl_G(\ddot{V}(H)) = \cap \{\ddot{V}(F);\ \ddot{V}(F) \text{ is preclosed subgraph}, \ddot{V}(H) \subseteq \ddot{V}(F)\}$, and the pre-interior subgraph is $pInt_G(\ddot{V}(H)) = \cup \{\ddot{V}(U);\ \ddot{V}(U) \text{ is preopen subgraph}, \ddot{V}(H) \subseteq \ddot{V}(U)\}$.

Definition 3.10

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$". Then the semi-preclosure subgraph is

$spCl_G(\ddot{\mathrm{V}}(H)) = \cap\{\ddot{\mathrm{V}}(\mathrm{F});\ \ddot{\mathrm{V}}(\mathrm{F})\text{is semi}-\text{preclosed subgraph}, \ddot{\mathrm{V}}(H) \subseteq \ddot{\mathrm{V}}(\mathrm{F})\}$, and the semi-preinterior subgraph is

$spInt_G(\ddot{\mathrm{V}}(H)) = \cup\{\ddot{\mathrm{V}}(\mathrm{U});\ \ddot{\mathrm{V}}(\mathrm{U})\text{is semi}-\text{preopen subgraph}, \ddot{\mathrm{V}}(H) \subseteq \ddot{\mathrm{V}}(\mathrm{U})\}$.

Definition 3.11

"Let $G(\ddot{\mathrm{V}},\ \ddot{\mathrm{E}})$ be a graph that contains a topological graph $(\ddot{\mathrm{V}}(G), \tau_G)$". Then the b-closure subgraph is

"$bCl_G(\ddot{\mathrm{V}}(H)) = \cap\{\ddot{\mathrm{V}}(\mathrm{F});\ \ddot{\mathrm{V}}(\mathrm{F})\text{is b}-\text{closed subgraph}, \ddot{\mathrm{V}}(H) \subseteq \ddot{\mathrm{V}}(\mathrm{F})\}$"

and the semi-prei-interior subgraph is

"$bInt_G(\ddot{\mathrm{V}}(H)) = \cup\{\ddot{\mathrm{V}}(\mathrm{U});\ \ddot{\mathrm{V}}(\mathrm{U})\text{is b}-\text{open subgraph}, \ddot{\mathrm{V}}(H) \subseteq \ddot{\mathrm{V}}(\mathrm{U})\}$".

4. Some results and application on (Semi, Pre, Semi-pre, b)-open subgraph.

Theorem 4.1

"Let $G(\ddot{\mathrm{V}},\ \ddot{\mathrm{E}})$ be a graph that contains a topological graph $(\ddot{\mathrm{V}}(G), \tau_G)$".

If H be a subgraph from G, then:

1) $sCl_G(\ddot{\mathrm{V}}(H)) = \ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$,

(2) $sInt_G(\ddot{\mathrm{V}}(H)) = \ddot{\mathrm{V}}(H) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H)))$.

(3) $pCl_G(\ddot{\mathrm{V}}(H)) = \ddot{\mathrm{V}}(H) \cup Cl_G(Int_G(\ddot{\mathrm{V}}(H)))$,

(4) $pInt_G(\ddot{\mathrm{V}}(H)) = \ddot{\mathrm{V}}(H) \cap Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$.

Proof

(1) Since $sCl_G(\ddot{\mathrm{V}}(H))$ is semi closed, we have $Int_G(Cl_G(sCl_G(\ddot{\mathrm{V}}(H)))) \subseteq sCl_G(\ddot{\mathrm{V}}(H))$.

Therefore, $Int_G(Cl_G(\ddot{\mathrm{V}}(H))) \subseteq sCl_G(\ddot{\mathrm{V}}(H))$, and hence $\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H))) \subseteq sCl_G(\ddot{\mathrm{V}}(H))$.

To establish the opposite inclusion we observe that $Int_G(\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H))))$

$= Int_G(Cl_G(\ddot{\mathrm{V}}(H))) \cup Cl_G(Int_G(Cl_G(\ddot{\mathrm{V}}(H))) \subseteq Cl_G(\ddot{\mathrm{V}}(H)) \cup Int_G(Cl_G(Int_G(\ddot{\mathrm{V}}(H))))$

$\subseteq Cl_G(\ddot{\mathrm{V}}(H)) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H))) = Cl_G(\ddot{\mathrm{V}}(H))$.

Thus $Int_G(Cl_G(\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H)))))$

$\subseteq Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$

$\subseteq V(H) \cup Int_G(Cl_G(V(H)))$.

Hence $\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$ is semi-closed, and so $sCl_G(\ddot{\mathrm{V}}(H)) \subseteq \ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$.

(2) Since $sInt_G(\ddot{\mathrm{V}}(H))$ is semi-open, we have $Cl_G(Int_G(sInt_G(\ddot{\mathrm{V}}(H)))) \subseteq sInt_G(\ddot{\mathrm{V}}(H))$.

Therefore, $Cl_G(Int_G(\ddot{\mathrm{V}}(H))) \subseteq sInt_G(\ddot{\mathrm{V}}(H))$, and hence $\ddot{\mathrm{V}}(H) \cup Cl_G(Int_G(\ddot{\mathrm{V}}(H))) \subseteq sInt_G(\ddot{\mathrm{V}}(H))$……(1)

To establish the opposite inclusion we observe that

$Cl_G(\ddot{\mathrm{V}}(H) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H)))) =$

$Int_G(Cl_G(\ddot{\mathrm{V}}(H))) \cap Int_G(Cl_G(Int_G(\ddot{\mathrm{V}}(H))))$

$\subseteq Int_G(\ddot{\mathrm{V}}(H)) \cap Int_G(Cl_G(Int_G\ddot{\mathrm{V}}(H)))$

$\subseteq Int_G(\ddot{\mathrm{V}}(H)) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H))) = Int_G(\ddot{\mathrm{V}}(H))$.

Thus $Cl_G(Int_G(\ddot{\mathrm{V}}(H) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H))))) \subseteq Cl_G(Int_G(\ddot{\mathrm{V}}(H)))$

$\subseteq \ddot{\mathrm{V}}(H) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H)))$……(2)

From (1) and (2) we get, $sInt_G(\ddot{\mathrm{V}}(H)) = \ddot{\mathrm{V}}(H) \cap Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$.

(3) Assume that, $\check{V}(H) \subseteq \check{V}(G)$, observe that

$Cl_G(Int_G(\check{V}(H) \cup Cl_G(Int_G(\check{V}(H)))))$

$\subseteq Cl_G(Int_G(\check{V}(H)) \cup Cl_G(Int_G(\check{V}(H))))$

$= Cl_G(Int_G(\check{V}(H)))$

$\subseteq \check{V}(H) \cup Cl_G(Int_G(\check{V}(H)))$. Hence

$\check{V}(H) \cup Cl_G(Int_G(\check{V}(H)))$ is

pre-closed and thus $pCl_G(\check{V}(H)) \subseteq \check{V}(H) \cup Cl_G(Int_G(\check{V}(H)))$

Conversely

Since,$pCl_G(\check{V}(H))$ is preclosed, we have

$Cl_G(Int_G(\check{V}(H)) \subseteq Cl_G(Int_G(pCl_G(\check{V}(H))))$

$\subseteq pCl_G(\check{V}(H))$,

and hence $\check{V}(H) \cup Cl_G(Int_G(\check{V}(H))) \subseteq pCl_G(\check{V}(H))$.

(4) Assume that, $\check{V}(H) \subseteq \check{V}(G)$, observe that

$Int_G(Cl_G(\check{V}(H) \cap Int_G(Cl_G(\check{V}(H)))))$

$\subseteq Int_G(Cl_G(\check{V}(H)) \cap Int_G(Cl_G(\check{V}(H))))$

$= Int_G(Cl_G(\check{V}(H)))$

$\subseteq \check{V}(H) \cap Int_G(Cl_G(\check{V}(H)))$. Hence $\check{V}(H) \cap Cl_G(Int_G(\check{V}(H)))$ is

pre-open and thus $pInt_G(\check{V}(H)) \subseteq \check{V}(H) \cap Int_G(Cl_G(\check{V}(H)))$…..(1)

Conversely

Since,$pInt_G(V(H))$ is preopen, we have

$Int_G(Cl_G(\check{V}(H))) \subseteq Int_G(Cl_G(pInt_G(\check{V}(H)))) \subseteq pInt_G(\check{V}(H))$,

and hence $\check{V}(H) \cap Int_G(Cl_G(\check{V}(H))) \subseteq pInt_G(\check{V}(H))$…..(2)

From (1) and (2) we get, $pInt_G(\check{V}(H)) = \check{V}(H) \cap Int_G(Cl_G(\check{V}(H)))$. ■

Theorem 4.2

"Let $G(\check{V}, \ddot{E})$ be a graph that contains a topological graph $(\check{V}(G), \tau_G)$".

If H be a subgraph from G, then:

(1)$spCl_G(\check{V}(H)) = \check{V}(H) \cup Int_G(Cl_G(Int_G(\check{V}(H))))$.

(2) $spInt_G(\check{V}(H)) = \check{V}(H) \cap Cl_G(Int_G(Cl_G(\check{V}(H))))$.

Proof

(1) We observe that, Assume that $\check{V}(H) \subseteq \check{V}(G)$. Then

$Int_G(Cl_G(Int_G(\check{V}(H)) \cup Int_G(Cl_G(Int_G(\check{V}(H))))))$

$\subset Int_G(Cl_G(Int_G(\check{V}(H) \cup Cl_G(Int_G(\check{V}(H))))))$

$\subset Int_G(Cl_G(Int_G(\check{V}(H)) \cup Cl_G(Int_G(\check{V}(H)))))$

$= Int_G(Cl_G(Int_G(\check{V}(H)))) \subset \check{V}(H) \subset Int_G(Cl_G(Int_G(\check{V}(H))))$. Hence

$\check{V}(H) \cup Int_G(Cl_G(Int_G(\check{V}(H))))$

Conversely

Since $spCl_G(\check{V}(H))$ is semi-preclosed subgraph, we have

$Int_G(Cl_G(Int_G(\check{V}(H))))$

$\subset Int_G(Cl_G(Int_G(spCl_G(\check{V}(H)))))$

$\subset spCl_G(\check{V}(H))$ and hence $\check{V}(H) \cup Int_G(Cl_G(Int_G(\check{V}(H)))) \subset spCl_G(\check{V}(H))$.

(2) We observe that,

$\check{V}(H) \cap Cl_G(Int_G(Cl_G(\check{V}(H))))$

$\subset Cl_G(Int_G(Cl_G(\check{V}(H))))$

$= Cl_G(Int_G(Cl_G(\check{V}(H))) \cap Int_G(Cl_G(\check{V}(H))))$

$\subset Cl_G(Int_G(Cl_G(\check{V}(H)) \cap Int_G(Cl_G(\check{V}(H)))))$

$\subset Cl_G(Int_G(Cl_G(\check{V}(H)) \cap Cl_G(Int_G(Cl_G(\check{V}(H))))))$. Hence

$\ddot{V}(H) \cap Cl_G(Int_G(Cl_G(\ddot{V}(H)))) \subset spInt_G(\ddot{V}(H))$.

Conversely

Since $spInt_G$ ($\ddot{V}(H)$) is semi-preopen subgraph, we have $spInt_G(\ddot{V}(H))$ $\subset Cl_G(Int_G(Cl_G(spInt_G(\ddot{V}(H)))))$ $\subset Cl_G(Int_G(Cl_G(\ddot{V}(H))))$ and hence $spInt_G(\ddot{V}(H))$ $\subset \ddot{V}(H) \cap Cl_G(Int_G(Cl_G(\ddot{V}(H))))$. ■

Theorem 4.3

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$". If H be a subgraph from G, then:

(1) $bCl_G(\ddot{V}(H)) = sCl_G(\ddot{V}(H)) \cap pCl_G(\ddot{V}(H))$.

(2) $bInt_G(\ddot{V}(H)) = sInt_G(\ddot{V}(H)) \cup pInt_G(\ddot{V}(H))$.

Proof

(1) Assume that $\ddot{V}(H) \subseteq \ddot{V}(G)$, since $bCl_G(\ddot{V}(H))$ is a b-closed subgraph, we have $bCl_G(\ddot{V}(H)) \supset Int_G(Cl_G(bCl_G(\ddot{V}(H)))) \cap Cl_G(Int_G(bCl_G(\ddot{V}(H))))$ $\supset Int_G(Cl_G(\ddot{V}(H))) \cap Cl_G(Int_G(\ddot{V}(H)))$ and so $bCl_G(\ddot{V}(H)) \supset \ddot{V}(H) \cup (Int_G(Cl_G(\ddot{V}(H))) \cap Cl_G(Int_G(\ddot{V}(H))))$ $= sCl_G(\ddot{V}(H)) \cap pCl_G(\ddot{V}(H))$. By theorem 4.1. The reverse inclusion is clear.

(2) Assume that $\ddot{V}(H) \subset \ddot{V}(G)$, since $bInt_G(\ddot{V}(H))$ is a b-open subgraph, we have $bInt_G(\ddot{V}(H)) \supset Int_G(Cl_G(bInt_G(\ddot{V}(H)))) \cup Cl_G(Int_G(bInt_G(\ddot{V}(H))))$ $\supset Int_G(Cl_G(\ddot{V}(H))) \cup Cl_G(Int_G(\ddot{V}(H)))$ and so $bInt_G(\ddot{V}(H)) \supset \ddot{V}(H) \cap Int_G(Cl_G(\ddot{V}(H))) \cup Cl_G(Int_G(\ddot{V}(H))))$ $= sInt_G(\ddot{V}(H)) \cup pInt_G(\ddot{V}(H))$. By theorem 4.1. The reverse inclusion is clear. ■

Theorem 4.4

"Let $G(\ddot{V}, \ddot{E})$ be a graph that contains a topological graph $(\ddot{V}(G), \tau_G)$". If H be a subgraph from G, then:

1) $sCl_G(sInt_G(\ddot{V}(H))) = sInt_G(\ddot{V}(H)) \cup Int_G(Cl_G(Int_G(\ddot{V}(H))))$.

(2) $pCl_G(pInt_G(\ddot{V}(H))) = pInt_G(\ddot{V}(H)) \cup Cl_G(Int_G(\ddot{V}(H)))$.

(3) $spCl_G(spInt_G(\ddot{V}(H)))$ $= spInt_G(spCl_G(\ddot{V}(H)))$.

Proof

(1) By theorem 4.1, we have

$sCl_G(sInt_G(\ddot{V}(H))) = sInt_G(\ddot{V}(H))$ $\cup Int_G(Cl_G(sInt_G(\ddot{V}(H))))$ $= sInt_G(\ddot{V}(H)) \cup Cl_G(Int_G(\ddot{V}(H)$ $\cap Cl_G(Int_G(\ddot{V}(H)))))$ $\subset sInt_G(\ddot{V}(H)) \cup Int_G(Cl_G(\ddot{V}(H))$ $\cap Cl_G(Int_G(\ddot{V}(H))))$ $= sInt_G(\ddot{V}(H)) \cup Int_G(Cl_G(Int_G(\ddot{V}(H))))$.

To establish the opposite inclusion we observe that

$sCl_G(sInt_G(\ddot{V}(H))) = sInt_G(\ddot{V}(H)) \cup Int_G(Cl_G(sInt_G(\ddot{V}(H))))$

$\supset sInt_G(\grave{\grave{V}}(H)) \cup Int_G(Cl_G(Int_G(\grave{\grave{V}}(H))))$.

(2) By theorem 4.1, we have

$pCl_G(pInt_G(\grave{\grave{V}}(H)))) = pInt_G(\grave{\grave{V}}(H))$

$\cup Cl_G(Int_G(pInt_G(\grave{\grave{V}}(H))))$

$= pInt_G(\grave{\grave{V}}(H)) \cup Cl_G(Int_G(\grave{\grave{V}}(H)))$.

(3) By theorem 4.1, we have

$spInt_G(spCl_G(\grave{\grave{V}}(H))) = spCl_G(\grave{\grave{V}}(H))$

$\cap Cl_G(Int_G(Cl_G(spCl_G(\grave{\grave{V}}(H)))))$

$= (\grave{\grave{V}}(H) \cup Int_G(Cl_G(Int_G(\grave{\grave{V}}(H)))))$

$\cap Cl_G(Int_G(Cl_G(\grave{\grave{V}}(H))))$

$= (\grave{\grave{V}}(H) \cap Cl_G(Int_G(Cl_G(\grave{\grave{V}}(H)))))$

$\cup Int_G(Cl_G(Int_G(\grave{\grave{V}}(H))))$

$= spInt_G(\grave{\grave{V}}(H))$

$\cup Int_G(Cl_G(Int_G(spInt_G(\grave{\grave{V}}(H)))))$

$= spInt_G(spCl_G(\grave{\grave{V}}(H)))$. ■

Theorem 4.5

"Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph that contains a topological graph $(\grave{\grave{V}}(G), \tau_G)$". If H be a subgraph from G; then

(1) $sInt_G(sCl_G(\grave{\grave{V}}(H))) = sCl_G(\grave{\grave{V}}(H)) \cap Cl_G(Int_G(Cl_G(\grave{\grave{V}}(H))))$.

(2) $pInt_G(pCl_G(\grave{\grave{V}}(H))) = pCl_G(\grave{\grave{V}}(H)) \cap Int_G(Cl_G(\grave{\grave{V}}(H)))$.

(3) $sCl_G(pCl_G(\grave{\grave{V}}(H))) = sCl_G(\grave{\grave{V}}(H)) \cup pCl_G(\grave{\grave{V}}(H))$.

Proof

(1) Assume that $\grave{\grave{V}}(H) \subseteq \grave{\grave{V}}(G)$ by theorem 4.1, we have

$sInt_G(sCl_G(\grave{\grave{V}}(H))) = sCl_G(\grave{\grave{V}}(H))$

$\cap Cl_G(Int_G(sCl_G(\grave{\grave{V}}(H))))$

$= sCl_G(\grave{\grave{V}}(H)) \cap Cl_G(Int_G(\grave{\grave{V}}(H)$

$\cup Int_G(Cl_G(\grave{\grave{V}}(H)))))$

$\supset sCl_G(\grave{\grave{V}}(H)) \cap Cl_G(Int_G(\grave{\grave{V}}(H))$

$\cup Int_G(Cl_G(\grave{\grave{V}}(H))))$

$= sCl_G(\grave{\grave{V}}(H)) \cap Cl_G(Int_G(Cl_G(\grave{\grave{V}}(H))))$.

To establish the opposite inclusion we observe that

$sInt_G(sCl_G(\grave{\grave{V}}(H))) = sCl_G(\grave{\grave{V}}(H))$

$\cap Cl_G(Int_G(Cl_G(\grave{\grave{V}}(H))))$

$= sCl_G(\grave{\grave{V}}(H)) \cap Cl_G(Int_G(Cl_G(\grave{\grave{V}}(H))))$.

(2) By theorem 4.1, we have

$pInt_G(pCl_G(\grave{\grave{V}}(H))) = pCl_G(\grave{\grave{V}}(H))$

$\cap Int_G(Cl_G(pCl_G(\grave{\grave{V}}(H))))$

$= pCl_G(\grave{\grave{V}}(H)) \cap Int_G(Cl_G(\grave{\grave{V}}(H)))$.

(3) By theorem 4.1, we have immediately

$sCl_G(pCl_G(\grave{\grave{V}}(H))) = sCl_G(\grave{\grave{V}}(H)) \cup Int_G(pCl_G(\grave{\grave{V}}(H)))$

$= sCl_G(\grave{\grave{V}}(H)) \cup Int_G(Cl_G(\grave{\grave{V}}(H)))$

$= sCl_G(\grave{\grave{V}}(H)) \cup pCl_G(\grave{\grave{V}}(H))$. ■

Corollary 4.6

"Let $G(\grave{\grave{V}}, \grave{\grave{E}})$ be a graph that contains a topological graph $(\grave{\grave{V}}(G), \tau_G)$". If H be a subgraph from G; then

$pInt_G(pCl_G(\grave{\grave{V}}(H))) \subseteq pCl_G(pInt_G(\grave{\grave{V}}(H)))$.

Proof

Assume that $\grave{\grave{V}}(H) \subseteq \grave{\grave{V}}(G)$ by theorem 4.1, we have

$pInt_G(pCl_G(\grave{\grave{V}}(H))) = pCl_G(\grave{\grave{V}}(H))$

$\cap Int_G(Cl_G(\grave{\grave{V}}(H)))$

$= (\grave{\grave{V}}(H) \cup Cl_G(Int_G(\grave{\grave{V}}(H))))$

$\cap Int_G(Cl_G(\grave{\grave{V}}(H)))$

$= (\grave{\grave{V}}(H) \cap Int_G(Cl_G(\grave{\grave{V}}(H))))$

$\cup (Cl_G(Int_G(\grave{\grave{V}}(H)))$

$\cap Int_G(Cl_G(\grave{\grave{V}}(H))))$

$\subseteq pInt_G(\grave{\grave{V}}(H)) \cup Cl_G(Int_G(\grave{\grave{V}}(H)))$

$= pCl_G(pInt_G(\grave{\grave{V}}(H)))$. ■

Corollary 4.7

"Let $G(\ddot{\mathrm{V}},\ \ddot{\mathrm{E}})$ be a graph that contains a topological graph $(\ddot{\mathrm{V}}(G), \tau_G)$". If H be a subgraph from G, then $pInt_G(pCl_G(\ddot{\mathrm{V}}(H))) \subseteq sInt_G(sCl_G(\ddot{\mathrm{V}}(H)))$.

Proof

Assume that $\ddot{\mathrm{V}}(H) \subseteq V(G)$ by theorem 4.4, we have

$pInt_G(pCl_G(\ddot{\mathrm{V}}(H))) = (\ddot{\mathrm{V}}(H) \cup Cl_G(Int_G(\ddot{\mathrm{V}}(H)))) \cap Int_G(Cl_G(\ddot{\mathrm{V}}(H))) = (\ddot{\mathrm{V}}(H) \cap Int_G(Cl_G(\ddot{\mathrm{V}}(H)))) \cup (Cl_G(Int_G(\ddot{\mathrm{V}}(H))) \cap Int_G(Cl_G(\ddot{\mathrm{V}}(H)) \subset (\ddot{\mathrm{V}}(H) \cap Cl_G(Int_G(Cl_G(\ddot{\mathrm{V}}(H))))) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H))) = (\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H)))) \cap Cl_G(Int_G(Cl_G(\ddot{\mathrm{V}}(H)))) = sInt_G(sCl_G(\ddot{\mathrm{V}}(H)))$. ■

Corollary 4.8

"Let $G(\ddot{\mathrm{V}},\ \ddot{\mathrm{E}})$ be a graph that contains a topological graph $(\ddot{\mathrm{V}}(G), \tau_G)$". If H be a subgraph from G, then $sCl_G(sInt_G(\ddot{\mathrm{V}}(H))) \subseteq spInt_G(spCl_G(\ddot{\mathrm{V}}(H))) \subseteq sInt_G(sCl_G(\ddot{\mathrm{V}}(H)))$.

Proof

By theorem 4.1, 4.2. We have

$sCl_G(sInt_G(\ddot{\mathrm{V}}(H))) = sInt_G(\ddot{\mathrm{V}}(H)) \cup Int_G(Cl_G(Int_G(\ddot{\mathrm{V}}(H)))) = (\ddot{\mathrm{V}}(H) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H)))) \cup Int_G(Cl_G(Int_G(\ddot{\mathrm{V}}(H)))) = (\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(Int_G(\ddot{\mathrm{V}}(H))))) \cap Cl_G(Int_G(\ddot{\mathrm{V}}(H))) \subseteq (\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(Int_G(\ddot{\mathrm{V}}(H))))) \cap Cl_G(Int_G(Cl_G(\ddot{\mathrm{V}}(H)))) = spInt_G(spCl_G(\ddot{\mathrm{V}}(H))) \subseteq (\ddot{\mathrm{V}}(H) \cup Int_G(Cl_G(\ddot{\mathrm{V}}(H)))) \cap Cl_G(Int_G(Cl_G(\ddot{\mathrm{V}}(H)))) = sInt_G(sCl_G(\ddot{\mathrm{V}}(H)))$. ■

Theorem 4.9

"Let $G(\ddot{\mathrm{V}},\ \ddot{\mathrm{E}})$ be a graph that contains a topological graph $(\ddot{\mathrm{V}}(G), \tau_G)$" in which every $\ddot{\mathrm{V}}(H)$ is preopen subgraph if and only if every open subgraph in $(\ddot{\mathrm{V}}(G), \tau_G)$ is closed subgraph.

Proof

Assume that $\ddot{\mathrm{V}}(A)$ be open subgraph. Then, $\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A) = Cl_G(V(G) - \ddot{\mathrm{V}}(A))$, which is preopen subgraph, so that $Cl_G(\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A)) \subset Int_G(Cl_G(Cl_G(\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A)))) = Int_G(Cl_G(\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A))) = Int_G(\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A))$. Thus $\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A) = Int_G(\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A))$, so that $\ddot{\mathrm{V}}(G) - \ddot{\mathrm{V}}(A)$ is open subgraph, $\ddot{\mathrm{V}}(G)$ is closed subgraph.

Conversely

Assume that $\ddot{\mathrm{V}}(H) \subseteq \ddot{\mathrm{V}}(G)$. Then $V(G) - Cl_G(\ddot{\mathrm{V}}(H))$ is open subgraph, and hence closed subgraph. Thus $\ddot{\mathrm{V}}(G) - Cl_G(\ddot{\mathrm{V}}(H)) = Cl_G(\ddot{\mathrm{V}}(G) - Cl_G(\ddot{\mathrm{V}}(H))) = \ddot{\mathrm{V}}(G) - Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$, so that $\ddot{\mathrm{V}}(H) \subseteq Cl_G(\ddot{\mathrm{V}}(H)) = Int_G(Cl_G(\ddot{\mathrm{V}}(H)))$, and hence $\ddot{\mathrm{V}}(H)$ is preopen subgraph. ■

Theorem 4.10

If $\check{V}(H)$ is semi-open subgraph then $\check{V}(H)$ is semi-preopen subgraph.

Proof

Suppose that $V(H)$ is semi-preopen, then $\check{V}(H) \subseteq Cl_G(Int_G(Cl_G(\check{V}(H))))$, $Int_G(Cl_G(\check{V}(H))) = Int_G(\check{V}(H))$. Which, implies $V(H) \subseteq Cl_G(Int_G(\check{V}(H)))$ that is $\check{V}(H) \in SP(\check{V}(G))$. ■

Theorem 4.11

If $\check{V}(H)$ is preopen subgraph then $\check{V}(H)$ is semi-preopen subgraph.

Proof

Since $V(H)$ is preopen subgraph and $\check{V}(H) \subseteq \check{V}(H)$, and since for any subgraph $\check{V}(H)$ from $\check{V}(G)$, $\check{V}(H) \subseteq Cl_G(\check{V}(H))$. Therefore, there exist preopen subgraph $\check{V}(H)$ such that $\check{V}(H) \subseteq \check{V}(H) \subseteq Cl_G(\check{V}(H))$. Thus $\check{V}(H)$ is semi-preopen. ■

Theorem 4.12

"Let $G(\check{V}, \check{E})$ be a graph that contains a topological graph $(\check{V}(G), \tau_G)$". If H be a subgraph from G, then the following are equivalent:

(1) $\check{V}(H)$ is b-open.

(2) $\check{V}(H) = pInt_G(\check{V}(H)) \cup sInt_G(\check{V}(H))$.

(3) $\check{V}(H) \subseteq pCl_G(pInt_G(\check{V}(H)))$.

Proof

(1) ⇒ (2) Assume that $\check{V}(H)$ is b-open, that is $\check{V}(H) \subseteq \check{V}(H) \subseteq Int_G(Cl_G(\check{V}(H))) \cup Cl_G(Int_G(\check{V}(H)))$. Then by Theorem 4.1, we have $pInt_G(\check{V}(H)) \cup sInt_G(\check{V}(H)) = (\check{V}(H) \cap Int_G(Cl_G(\check{V}(H)))) \cup (\check{V}(H) \cap Cl_G(Int_G(\check{V}(H)))) = \check{V}(H) \cap (Int_G(Cl_G(\check{V}(H))) \cup Cl_G(Int_G(\check{V}(H)))) = \check{V}(H)$.

(2) ⇒ (3) By theorem 4.1, we have

$\check{V}(H) = pInt_G(\check{V}(H)) \cup sInt_G(\check{V}(H))$

$= pInt_G(\check{V}(H)) \cup (\check{V}(H) \cap Cl_G(Int_G(\check{V}(H))))$

$\subseteq pInt_G(\check{V}(H)) \cup Cl_G(Int_G(\check{V}(H)))$

$= pCl_G(pInt_G(\check{V}(H)))$.

(3) ⇒ (1) By theorem 4.1, we have

$V(H) \subseteq pCl_G(pInt_G(\check{V}(H))) = pInt_G(\check{V}(H))$

$\cup\ Cl_G(Int_G(\check{V}(H))) \subset Int_G(Cl_G(\check{V}(H))) \cup Cl_G(Int_G(\check{V}(H)))$. ■

Theorem 4.13

"Let $G(\check{V}, \check{E})$ is a graph that contains a topological graph $(\check{V}(G), \tau_G)$".

If H be a subgraph from G, then the following are equivalent:

(1) $\check{V}(H) \in SPO(\check{V}(G))$.

(2) $\check{V}(H) \subseteq spInt_G(spCl_G(\check{V}(H)))$.

(3) $\check{V}(H) \subseteq sInt_G(sCl_G(\check{V}(H)))$.

Proof

(1) ⇒ (2) Assume that $\check{V}(H)$ is semi-preopen subgraph. Then

$\check{V}(H) = spInt_G(\check{V}(H))$

$\subseteq spInt_G(spCl_G(\check{V}(H)))$.

(2) ⇒ (3) This follows immediately from Corollary 4.7.

(3) ⇒ (1) Assume that $\check{V}(H) \subseteq sInt_G(sCl_G(\check{V}(H)))$

$= sCl_G\ddot{V}(H)) \cap Cl_G(Int_G(Cl_G(\ddot{V}(H))))$.

This implies $\ddot{V}(H) \subseteq Cl_G(Int_G(Cl_G(\ddot{V}(H))))$ and therefore, $\ddot{V}(H) \in SPO(\ddot{V}(G))$. ■

Example 4.14

Let $G(\ddot{V}, \ddot{E})$ be a graph (see figure 3), if H be a subgraph from G with vertices $\ddot{V}(H) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$. Then $\ddot{v}_1R = \{\ddot{v}_4, \ddot{v}_5, \ddot{v}_6, \ddot{v}_7\}$, $\ddot{v}_2R = \{\ddot{v}_5, \ddot{v}_6\}$, $\ddot{v}_3R = \{\ddot{v}_4, \ddot{v}_5, \ddot{v}_7\}$, $\ddot{v}_4R = \{\ddot{v}_1, \ddot{v}_3\}$, $\ddot{v}_5R = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_7\}$, $\ddot{v}_6R = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_7\}$, $\ddot{v}_7R = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_5, \ddot{v}_6\}$. So $\ddot{v}_2R \cap \ddot{V}(H) \neq \emptyset$, $\ddot{v}_5R \cap \ddot{V}(H) \neq \emptyset$, $\ddot{v}_7R \cap \ddot{V}(H) \neq \emptyset$. $Cl_G(\ddot{V}(H)) = \ddot{V}(H) \cup \{\ddot{v}_2, \ddot{v}_5, \ddot{v}_7\} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4, \ddot{v}_5, \ddot{v}_6, \ddot{v}_7\}$.

$Int_G(\ddot{V}(H)) = \{v_4\}$, $Cl_G(Int_G(\ddot{V}(H))) = \{v_1, v_3, v_4\}$,

$Int_G(Cl_G(\ddot{V}(H))) = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4, \ddot{v}_5, \ddot{v}_6, \ddot{v}_7\}$,

$Int_G(Cl_G(Int_G(\ddot{V}(H)))) = \{\ddot{v}_4\}$,

$Cl_G(Int_G(Cl_G(\ddot{V}(H)))) = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4, \ddot{v}_5, \ddot{v}_6, \ddot{v}_7\}$. Then

$sCl_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4, \ddot{v}_5, \ddot{v}_6, \ddot{v}_7\}$,

$sInt_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4\}$.

$pCl_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$,

$pInt_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$.

$spCl_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$,

$spInt_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$.

$bCl_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$,

$bInt_G(\ddot{V}(H)) = \{\ddot{v}_1, \ddot{v}_3, \ddot{v}_4, \ddot{v}_6\}$.

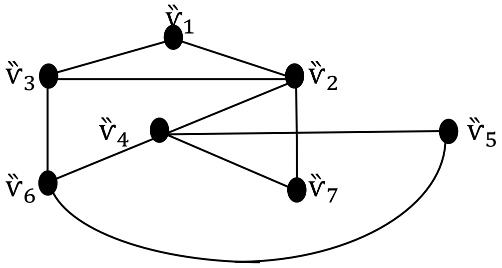

Figure 3. A simple graph G.

Conclusion

In this paper, we were can topological construct of any graph by using the definition of topological graph. We also, studied graph closure and graph interior. So, we introduced (Semi, Pre, Semi-Pre, b)-open subgraph with some result and example.

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