Some Relations On Fuzzy δ-Open Set in Fuzzy Topological Space on Fuzzy Set

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Abstract : The aim of introduce and study the notion of a fuzzy δ-open set (Ω-open set, α − Ω-open set, feebly –open set, α-open set, β-open set, Sp-open set, a-open set) and the relationships between them and fuzzy δ-open set in fuzzy topological space on fuzzy set and some properties, remarks related to them .

***Keywords*: fuzzy open set, membership function, fuzzy topology**

1. Introduction :

The recent concept is introduced by Zadeh in (1965) [18], In (1968) Chang [4] introduced the definition of fuzzy topological spaces and extended in a straight forward manner some concepts of crisp topological spaces to fuzzy topological spaces. In (1973) wrong given The definition of fuzzy point such away that an ordinary point was not special case of fuzzy point.

In (1974) While Wong [17] discussed and generalized some properties of fuzzy topological spaces. In (1980) Ming, p.p. and Ming, L.Y. [13] used fuzzy topology to define the neighborhood structure of fuzzy point.

In (1982) Hdeib [7] introduced the concept of fuzzy Ω-open set in topological space, In (1982) Maheshwari S.N. and Jain P.G. [9] defined the notion of fuzzy feebly open and fuzzy feebly closed set in fuzzy topological space and studied their properties .

In (1986) Mashhour A.S. and others [12] introduced the notion of α-open sets in topological space. In (1987) Mashhour A.S. and others [11] and in (1991) A.S.Bin Shahana [1] in introduced the concept fuzzy β-open set in general topology, In (1995) A.M.Zahran [2] introduced the notion of fuzzy δ-open set in fuzzy topological spaces, In (1996) Dontchev and Przemski have introduced the concept of Sp-open set in general topology [5].

In (1998) Bai Shi – Zhong and Wang Wan – Liang [3] have introduced The notion of fuzzy topology on fuzzy set and they defined the quasi-coincident in fuzzy topological space on fuzzy set. In (2003) Mahmoud, fath-Alla and Abd.Ellah [10] defined fuzzy interior and fuzzy closure in fuzzy topological space on fuzzy set and investigate their properties , In (2016) otchana and others introduced the concept of α-Ω open set in topological space [14].

1.1 Definition [4] :

Let X be a nonempty set, a fuzzy set Ã in X is characterized by a function

$\mu_{\tilde{A}} : X \to I$, where $I = [0,1]$ which is written as

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \le \mu_{\tilde{A}}(x) \le 1\}$, the collection of all fuzzy sets in X will be denoted by I^X,that is

$I^X = \{ \tilde{A} : \tilde{A}$ is a fuzzy sets in X$\}$ where $\mu_{\tilde{A}}$ is called the membership function

1.2 Proposition [18] **:**

Let Ã and B̃ be two fuzzy sets in X with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively, then for all x ∈ X: -

1. $\tilde{A} \subseteq \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$.
2. $\tilde{A} = \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.
3. $\tilde{C} = \tilde{A} \cap \tilde{B} \leftrightarrow C(x) = \min\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.
4. $\tilde{D} = \tilde{A} \cup \tilde{B} \leftrightarrow D(x) = \max\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.
5. $\tilde{B}^c$ the complement of B̃ with membership function $\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)$.

***1.3 Definition [10]*:**

A fuzzy point x_r is a fuzzy set such that :

$\mu_{x_r}(y) = r > 0$ if $x = y$, $\forall y \in X$ and

$\mu_{x_r}(y) = 0$ if $x \ne y$, $\forall y \in X$

The family of all fuzzy points of Ã will be denoted by FP(Ã) .

***1.4 Definition [4]*:**

A collection T̃ of a fuzzy subsets of Ã, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on Ã if it satisfied the following conditions

1. $\tilde{A}, \tilde{\phi} \in \tilde{T}$
2. If $\tilde{B}, \tilde{C} \in \tilde{T}$, then $\tilde{B} \cap \tilde{C} \in \tilde{T}$
3. If $\tilde{B}_\alpha \in \tilde{T}$, then $\cup_\alpha \tilde{B}_\alpha \in \tilde{T}$, $\alpha \in \Lambda$

(Ã , T̃) is said to be Fuzzy topological space and every member of T̃ is called fuzzy open set in Ã and its complement is a fuzzy closed set.

***1.5 Definition [8]*:**

A fuzzy set B̃ in a fuzzy topological space (Ã ,T̃) is said to be

Fuzzy delta set if, $\mu_{Int(Cl(\tilde{B}))}(x) \le \mu_{\tilde{B}}(x) \le \mu_{Cl(Int(\tilde{B}))}(x)$

Such that,

- **Fuzzy δ-open set if** $\mu_{Int(Cl(\tilde{B}))}(x) \le \mu_{B}(x)$.
- **Fuzzy δ-closed set if** $\mu_{B}(x) \le \mu_{Cl(Int(\tilde{B}))}(x)$.
- **Fuzzy δ-closed set if A= δcl(A)** , where $\mu_{\delta cl(\tilde{B})}(x) = \min\{ \mu_F(x) : \tilde{F}$ **is a fuzzy δ – closed set in Ã ,** $\mu_B(x) \le \mu_F(x)\}$.

  The complement of fuzzy δ-closed set is fuzzy δ-open set

1.6 Remark:

1. The family of all fuzzy δ-open sets in a fuzzy topological space (Ã ,T̃) will be denoted by FδO(Ã).
2. The family of all fuzzy δ-closed sets in a fuzzy topological space (Ã ,T̃) will be denoted by FδC(Ã).

***1.7 Proposition [13]*:**

1) Any union of fuzzy δ-open sets in a fuzzy topological space (Ã,T̃) is a fuzzy δ-open set in Ã.
2) Any intersection of fuzzy δ-closed sets in a fuzzy topological space (Ã,T̃) is a fuzzy δ-closed set in Ã.

Remark 1.8 **:**

(1) The intersection of two fuzzy δ-open sets is not necessary fuzzy δ-open set.

(2) The union of two fuzzy δ-closed sets is not necessary fuzzy δ-closed set.

As shown by the following example :-

1.9 Example:

Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{K}$, $\tilde{L}$ be fuzzy subsets of $\tilde{A}$ where:

$\tilde{A}$ = {(a , 0.4) , (b , 0.6)} ,

$\tilde{B}$ = {(a , 0.0) , (b ,0.4)} ,

$\tilde{C}$ = {(a , 0.4) , (b , 0.0)}

$\tilde{D}$ = {(a , 0.4) , (b ,0.4)} ,

$\tilde{E}$ = {(a , 0.0) , (b , 0.2)} ,

$\tilde{F}$ = {(a , 0.4) , (b ,0.2)} ,

$\tilde{K}$ = {(a , 0.0) , (b , 0.5)} ,

$\tilde{L}$= {(a , 0.4) , (b ,0.1)} ,

The fuzzy topology defined on $\tilde{A}$ is

$\tilde{T}$ = { $\tilde{\emptyset}$, $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{K}$, $\tilde{L}$}

$\tilde{K}$, $\tilde{L}$ are fuzzy δ-open sets but $\tilde{K} \cap \tilde{L}$ is not fuzzy δ-open set also $\tilde{K}^c \cup \tilde{L}^c$ is not fuzzy δ-closed set.

1.10 proposition [15]:

Every fuzzy δ-open set (fuzzy δ-closed set) is fuzzy open set
(fuzzy closed set).

Proof : Obvious

1.11 Remark:

The converse of proposition(1.10) is not true in general as the following example shows:-

1.12 Example:

Let X = { a , b , c } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\tilde{H}$ be fuzzy subsets

of $\tilde{A}$ where:

$\tilde{A}$ = {(a ,0.9) ,(b ,0.9) , (c ,0.9)} ,

$\tilde{B}$ = {(a ,0.2) , (b ,0.2) , (c ,0.3)} ,

$\tilde{C}$ = {(a ,0.3) , (b , 0.2) , (c ,0.3)} ,

$\tilde{D}$ = {(a ,0.4) , (b ,0.4) , (c ,0.3)} ,

$\tilde{E}$ = {(a ,0.5) , (b ,0.5) , (c ,0.6)} ,

$\tilde{F}$ = {(a ,0.3) , (b ,0.3) , (c ,0.3)} ,

$\tilde{G}$ = {(a ,0.5) , (b ,0.7) , (c ,0.6)} ,

$\tilde{H}$ = {(a ,0.1) , (b ,0.3) , (c ,0.3)} ,

The fuzzy set $\tilde{B}$ in fuzzy topological space ($\tilde{A}$,$\tilde{T}$) is fuzzy open set [fuzzy closed set] but not fuzzy δ-open set [fuzzy δ-closed set]

1.13 Definition [9]:

A fuzzy set $\tilde{B}$ in a fuzzy topological space ($\tilde{A}$,$\tilde{T}$) is called fuzzy δ-neighborhood (δnbhd) of a fuzzy point x_r in $\tilde{A}$ if there is a fuzzy δ-open set $\tilde{G}$ in $\tilde{A}$ such that

$\mu_{x_r}(x) \leq \mu_{\tilde{G}}(x) \leq \mu_{\tilde{B}}(x), \ \forall x \in X.$

1.14 Proposition:

Every fuzzy δ-neighborhood $\tilde{B}$ of x_r is fuzzy neighborhood.

Proof : Obvious

1.15 Definition [9]:

A fuzzy set $\tilde{B}$ in a fuzzy topological space ($\tilde{A}$,$\tilde{T}$) is called fuzzy δ-quasi neighborhood of a fuzzy point x_r in $\tilde{A}$ if there is a fuzzy δ-open set $\tilde{G}$ in $\tilde{A}$ such that $x_r \, q \, \tilde{G}$ and $\mu_{\tilde{G}}(x) \leq \mu_{\tilde{B}}(x)$, $\forall x \in X$.

1.16 Proposition:

Let $\tilde{B}$ be a fuzzy set in a fuzzy topological space ($\tilde{A}$,$\tilde{T}$), then the following statements are equivalent ;

1) $\tilde{B}$ is fuzzy δ-open set in $\tilde{A}$.
2) $\tilde{B}$ is a fuzzy δ-neighborhood of x_r, for each fuzzy point x_r in $\tilde{B}$
3) For each fuzzy point x_r in $\tilde{B}$, there exist a fuzzy δ-neighborhood $\tilde{C}$ of x_r such that $\mu_{x_r}(x) \leq \mu_{\tilde{C}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{B}}(x)$, $\forall x \in X$

Proof: Obvious

1.17 Definition:

A fuzzy set $\tilde{B}$ in ($\tilde{A}$,$\tilde{T}$) is said to be fuzzy δ-clopen set if and only if both fuzzy δ-open set and fuzzy δ-closed set.

Definition 1.18 :

Let $\tilde{B}$ be a fuzzy set in a fuzzy topological space ($\tilde{A}$,$\tilde{T}$) then :

- **The δ – closure** of $\tilde{B}$ is denoted by ($\delta cl(\tilde{B})$) and defined by

$\mu_{\delta cl(\tilde{B})}(x)$ = min{ $\mu_{\tilde{F}}(x)$: $\tilde{F}$ is a fuzzy δ – closed set in $\tilde{A}$, $\mu_{\tilde{B}}(x) \le \mu_{\tilde{F}}(x)$} .

- **The δ – interior** of $\tilde{B}$ is denoted by ($\delta int(\tilde{B})$) and defined by

$\mu_{\delta int(\tilde{B})}(x)$ = max { $\mu_{\tilde{G}}(x)$: $\tilde{G}$ is a fuzzy δ – open set in $\tilde{A}$, $\mu_{\tilde{G}}(x) \le \mu_{\tilde{B}}(x)$}.

1.19 Theorem:

Let $\tilde{B}$ and $\tilde{C}$ be fuzzy sets in a fuzzy topological space ($\tilde{A}$, $\tilde{T}$), then ;

1. $\mu_{\delta cl(\tilde{\emptyset})}(x) = \mu_{\tilde{\emptyset}}(x)$ and $\mu_{\delta cl(\tilde{A})}(x) = \mu_{\tilde{A}}(x)$.
2. If $\mu_{\tilde{B}}(x) \le \mu_{\tilde{C}}(x)$ then $\mu_{\delta cl(\tilde{B})}(x) \le \mu_{\delta cl(C)}(x)$.
3. $\mu_{\tilde{B}}(x) \le \mu_{\delta cl(\tilde{B})}(x)$.
4. $\mu_{\delta cl(\delta cl(\tilde{B}))}(x) = \mu_{\delta cl(\tilde{B})}(x)$.
5. $\mu_{\delta cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x)\})}(x) \le$ min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\delta cl(C)}(x)$ }.
6. $\mu_{\delta cl(max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x))}(x)$ = max { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\delta cl(C)}(x)$ }.

1.20 Theorem:

Let $\tilde{B}$ and $\tilde{C}$ be fuzzy sets in a fuzzy topological space ($\tilde{A}$, $\tilde{T}$), then ;

1. $\mu_{\delta int(\tilde{\emptyset})}(x) = \mu_{\tilde{\emptyset}}(x)$ and $\mu_{\delta int(\tilde{A})}(x) = \mu_{\tilde{A}}(x)$.
2. If $\mu_{\tilde{B}}(x) \le \mu_{\tilde{C}}(x)$, then $\mu_{\delta int(\tilde{B})}(x) \le \mu_{\delta int(C)}(x)$.
3. $\mu_{\delta int(\tilde{B})}(x) \le \mu_{\tilde{B}}(x)$.
4. $\mu_{\delta int(\delta int(\tilde{B}))}(x) = \mu_{\delta int(\tilde{B})}(x)$.
5. $\mu_{\delta int(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x))}(x)$ = min { $\mu_{\delta int(\tilde{B})}(x)$, $\mu_{\delta int(C)}(x)$ }.
6. max { $\mu_{\delta int(\tilde{B})}(x)$, $\mu_{\delta int(C)}(x)$ } $\le \mu_{\delta int(max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x))}(x)$.

Proof : Obvious

***1.21 Theorem* [15]:**

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space and $\tilde{B}$ are fuzzy set in $\tilde{A}$ and x_r is a fuzzy point in $\tilde{A}$, then $\mu_{x_r}(x) \le \mu_{\delta int(\tilde{B})}(x)$ if and only if x_r has a fuzzy δ-neighborhood contained in $\tilde{B}$.

Proof : Obvious

1.22 Definition [4]:

A collection **G** = {$\tilde{G}_\alpha$: $\alpha \in \Lambda$ } of fuzzy open sets in ($\tilde{A}$, $\tilde{T}$) is said to be fuzzy open cover of a fuzzy set $\tilde{B}$ of $\tilde{A}$ if $\mu_{\tilde{A}}(x) \le$ sup { $\mu_{\tilde{U}_\alpha}(x)$: $\alpha \in \Lambda$ }

Definition 1.23 [16]:

A collection {$\tilde{B}_\alpha$: $\alpha \in \Lambda$ } of a fuzzy sets in ($\tilde{A}$, $\tilde{T}$) is said to be fuzzy locally finite if for every $\mu_{x_r}(x) \le \mu_{\tilde{A}}(x)$, there exist a fuzzy neighborhood $\tilde{N}$ of x_r which is quasi coincident with at most a finite number of the members of {$\tilde{B}_\alpha$: $\alpha \in \Lambda$ } .

1.24 Definition [16]:

Let B = {$\tilde{B}_\alpha$: $\alpha \in \Lambda$ } , **C** = {$\tilde{C}_\beta$: $\beta \in \Lambda$ } ($\beta < \alpha$) be any two collection of fuzzy sets in ($\tilde{A}$, $\tilde{T}$) , then C is a refinement of B if for each $\beta \in \Lambda$ there exist $\alpha \in \Lambda$ such that $\mu_{\tilde{C}_\beta}(x) \le \mu_{\tilde{B}_\alpha}(x)$

2. Some Types of Fuzzy Open Sets:

In this section we study the properties and relations of various types of fuzzy open set in fuzzy topological spaces on fuzzy set which will be needed later on

2.1 Definition:

A fuzzy set $\tilde{B}$ of a fuzzy topological space ($\tilde{A}$,$\tilde{T}$) is said to be :-

1) **Fuzzy Ω-open (Fuzzy Ω-closed)** set if

$\mu_{Cl(\tilde{B})}(x) \le \mu_{Cl(Int(\tilde{B}))}(x)$,

($\mu_{intCl(\tilde{B})}(x) \le \mu_{cl(\tilde{B})}(x)$) , $\forall$ x $\in$ X.

The family of all fuzzy Ω-open (fuzzy Ω-closed) sets

in Ã will be denoted by FΩO(Ã) (FΩC(Ã)).

2) **Fuzzy $\boldsymbol{\alpha} - \Omega$ open (Fuzzy $\boldsymbol{\alpha} - \Omega$ closed**) set if

$\mu_{\tilde{B}}(x) \leq \mu_{Int_{\Omega}(Cl(Int_{\Omega}(\tilde{B})))}(x)$, $(\mu_{Cl_{\Omega}(Int(Cl_{\Omega}(\tilde{B})))} \leq \mu_{\tilde{B}}(x))$.

B̃ is called **(**Fuzzy $\alpha - \Omega$ closed **)** set if its complement is Fuzzy

$\alpha - \Omega$open sets

the family of all Fuzzy $\alpha - \Omega$ open (Fuzzy $\alpha - \Omega$ closed) sets

in Ã will be denoted by F$\alpha - \Omega$O(Ã) (F$\alpha - \Omega$C(Ã)).

3) **Fuzzy $\boldsymbol{feebly - open}$ ($\boldsymbol{feebly - closed}$)** set if

$\mu_{\tilde{B}}(x) \leq \mu_s(Cl(Int(\tilde{B})))(x)$, $(\mu_s(Int(Cl(\tilde{B})))(x) \leq \mu_{\tilde{B}}(x)$, $\forall x \in X$

The family of all fuzzy $feebly$ – open (fuzzy $feebly$ – closed) sets in Ã will be denoted by F$feebly$O(Ã) (F$feebly$C(Ã)).

4) **Fuzzy $\boldsymbol{\alpha}$-open (fuzzy $\boldsymbol{\alpha}$-closed)** set if

$\mu_{\tilde{B}}(x) \leq \mu_{Int(Cl(Int(\tilde{B})))}(x)$, $(\mu_{Cl(Int(Cl(\tilde{B})))} \leq \mu_{\tilde{B}}(x))$.

The family of all fuzzy α-open (fuzzy α-closed) sets in Ã will be denoted by FαO(Ã) (FαC(Ã)).

5) **Fuzzy β-open (fuzzy β-closed)** set if

$\mu_{\tilde{B}}(x) \leq \mu_{Int(Cl(\tilde{B}))}(x)$, $(\mu_{Cl(Int(\tilde{B}))} \leq \mu_{\tilde{B}}(x))$, $\forall x \in X$

The family of all fuzzy β-open (fuzzy β-closed) sets in Ã will be denoted by FβO(Ã) (FβC(Ã)).

6) **Fuzzy Sp-open (fuzzy Sp-closed)** set if

$\mu_{\tilde{B}}(x) \leq \max\{\mu_{Int(Cl(\tilde{B}))}(x), \mu_{Cl(Int(\tilde{B}))}(x)\}$

$\mu_{\tilde{B}}(x) \geq \min\{\mu_{Int(Cl(\tilde{B}))}(x), \mu_{Cl(Int(\tilde{B}))}(x)\}$, $\forall x \in X$

The family of all fuzzy Sp-open (fuzzy Sp-closed) sets in Ã will be denoted by FSpO(Ã) (FSpC(Ã)).

7) **Fuzzy a-open (fuzzy a-closed)** set if ,

$\mu_{\tilde{B}}(x) \leq \mu_{Int(Cl(Int_s(\tilde{B})))}(x)$, $(\mu_{Cl(Int(Cl_s(\tilde{B})))} \leq \mu_{\tilde{B}}(x))$

The family of all fuzzy a-open (fuzzy a-closed) sets in Ã will be denoted by FaO(Ã) (FaC(Ã)).

2.2 Definition:

Let B̃ is a fuzzy set in a fuzzy topological space (Ã ,T̃) then :

- **The** Ω– **closure** of B̃ is denoted by (Ωcl(B̃)) and defined by $\mu_{\Omega cl(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy Ω – closed set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$
- **The $\boldsymbol{\alpha} - \Omega$– closure** of B̃ is denoted by ($\boldsymbol{\alpha} - \Omega$cl(B̃)) and defined by $\mu_{\alpha - \Omega cl(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy $\boldsymbol{\alpha} - \Omega$ closed set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$
- **The $\boldsymbol{feebly}$– closure** of B̃ is denoted by ($feebly$cl(B̃)) and defined by $\mu_{feebly cl(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy $feebly$ – closed set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$
- **The $\boldsymbol{\alpha}$– closure** of B̃ is denoted by (αcl(B̃)) and defined by $\mu_{\alpha cl(\tilde{B})}(x) = \min\{\mu_{cl(\tilde{F})}(x) : \tilde{F}$ is a fuzzy open set in Ã, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$

2.3 Proposition:

Let (Ã , T̃) be a fuzzy topological space then :

1) The complement of fuzzy Ω-open set is fuzzy Ω-closed set .
2) The complement of fuzzy α-Ω -open set is fuzzy α-Ω -closed set
3) The complement of fuzzy feebly-open set is fuzzy feebly-closed set
4) The complement of fuzzy α-open set is fuzzy α-closed set
5) The complement of fuzzy β-open set is fuzzy β-closed set
6) The complement of fuzzy Sp -open set is fuzzy Sp -closed set
7) The complement of fuzzy a -open set is fuzzy a -closed set

Proof **:** Obvious .

2.4 Proposition:

Let (Ã , T̃) be a fuzzy topological space then :

1) Every fuzzy δ-open set is fuzzy open set (fuzzy Ω-open set , fuzzy feebly-open set , fuzzy a-open set)
2) Every fuzzy open set is fuzzy Ω-open set (fuzzy feebly open set , fuzzy α-open set)
3) Every fuzzy Ω-open set is fuzzy α-Ω open set (fuzzy a-open set)
4) Every fuzzy α-open set is fuzzy α-Ω open set (fuzzy β-open set , fuzzy Sp-open set)
5) Every fuzzy β-open set is fuzzy Sp-open set.
6) Every fuzzy a-open set is fuzzy α-open set.

2.5 Remark:

The converse of proposition (1.3.4) is not true in general as following example shows:-

2.6 Examples:

1. Let X = { a , b} and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\tilde{H}$, $\tilde{J}$, $\tilde{K}$ are fuzzy subset in $\tilde{A}$ where
   $\tilde{A}$ = { (a , 0.6) , (b , 0.6) } ,
   $\tilde{B}$ = { (a , 0.6) , (b , 0.0) }
   $\tilde{C}$ = { (a , 0.0) , (b , 0.4) } ,
   $\tilde{D}$ = { (a , 0.6) , (b , 0.4) }
   $\tilde{E}$ = { (a , 0.1) , (b , 0.5) } ,
   $\tilde{F}$ = { (a , 0.6) , (b , 0.5) }
   $\tilde{G}$ = { (a , 0.1) , (b , 0.0) } ,
   $\tilde{H}$ = { (a , 0.1) , (b , 0.4) }
   $\tilde{J}$ = { (a , 0.0) , (b , 0.5) }
   $\tilde{K}$ = { (a , 0.5) , (b , 0.0) }

Then the fuzzy set $\tilde{H}$ is a fuzzy Ω-open set (fuzzy open set) but not fuzzy δ – open set , also $\tilde{C}$, $\tilde{G}$ are fuzzy open set but not fuzzy δ-open set , $\tilde{H}$ is fuzzy feebly open set but not fuzzy δ-open set.
Also , the fuzzy set $\tilde{E}$ is a fuzzy α-Ω-open set but not fuzzy α – open set (not fuzzy Ω-open set) ,
The fuzzy set $\tilde{H}$, $\tilde{K}$ are fuzzy α-open set but not fuzzy a-open set .
The fuzzy set $\tilde{C}$, $\tilde{G}$ are fuzzy Sp-open set but fuzzy feebly open set
(not fuzzy α-open set) , also they are fuzzy β-open set but not fuzzy α-open set .

2. Let X={ a , b } and , $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$, $\tilde{B}_5$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_8$, $\tilde{B}_9$, $\tilde{B}_{10}$, $\tilde{B}_{11}$, $\tilde{B}_{12}$, $\tilde{B}_{13}$, $\tilde{B}_{14}$, $\tilde{B}_{15}$, $\tilde{B}_{16}$, $\tilde{B}_{17}$, are fuzzy subset of $\tilde{A}$ where:
   $\tilde{A}$ = {(a , 0.7) , (b , 0.7)} ,
   $\tilde{B}_2$ = {(a , 0.0) , (b , 0.7)} ,
   $\tilde{B}_3$ = {(a , 0.3) , (b , 0.7)} ,
   $\tilde{B}_4$ = {(a , 0.0) , (b , 0.4)},
   $\tilde{B}_5$ = {(a , 0.6) , (b , 0.7)} ,
   $\tilde{B}_6$ = {(a , 0.0) , (b , 0.2)} ,
   $\tilde{B}_7$ = {(a , 0.4) , (b , 0.0)} ,
   $\tilde{B}_8$ = {(a , 0.4) , (b , 0.5)} ,
   $\tilde{B}_9$ = {(a , 0.7) , (b , 0.0)} ,
   $\tilde{B}_{10}$ = {(a , 0.7) , (b , 0.3)} ,
   $\tilde{B}_{11}$= {(a , 0.6) , (b , 0.2)} ,
   $\tilde{B}_{12}$= {(a , 0.4) , (b , 0.4)} ,

$\tilde{B}_{13}$= {(a , 0.3) , (b , 0.2)} ,
$\tilde{B}_{14}$= {(a , 0.4) , (b , 0.1)} ,
$\tilde{B}_{15}$= {(a , 0.7) , (b , 0.4)} ,
$\tilde{B}_{16}$= {(a , 0.4) , (b , 0.7)} ,
$\tilde{B}_{17}$= {(a , 0.7) , (b , 0.2)}

Then the fuzzy set $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_9$, $\tilde{B}_{15}$ is a fuzzy Sp-open set but not fuzzy $feebly$ – open set , also $\tilde{B}_8$, $\tilde{B}_{12}$ are fuzzy α-open set but not fuzzy a-open set , $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_9$, $\tilde{B}_{11}$, $\tilde{B}_{13}$, $\tilde{B}_{14}$, $\tilde{B}_{15}$, $\tilde{B}_{17}$ is fuzzy β- open set but not fuzzy α-open set.

Also , the fuzzy set $\tilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_9$, $\tilde{B}_{15}$ is a fuzzy Sp-open set but not fuzzy α – open set,

The fuzzy set $\tilde{B}_8$ are fuzzy Sp-open set but not fuzzy β-open set .

2.7 Remark:

Figure - 1 – illustrates the relation between fuzzy δ-open set and some types of fuzzy open sets.

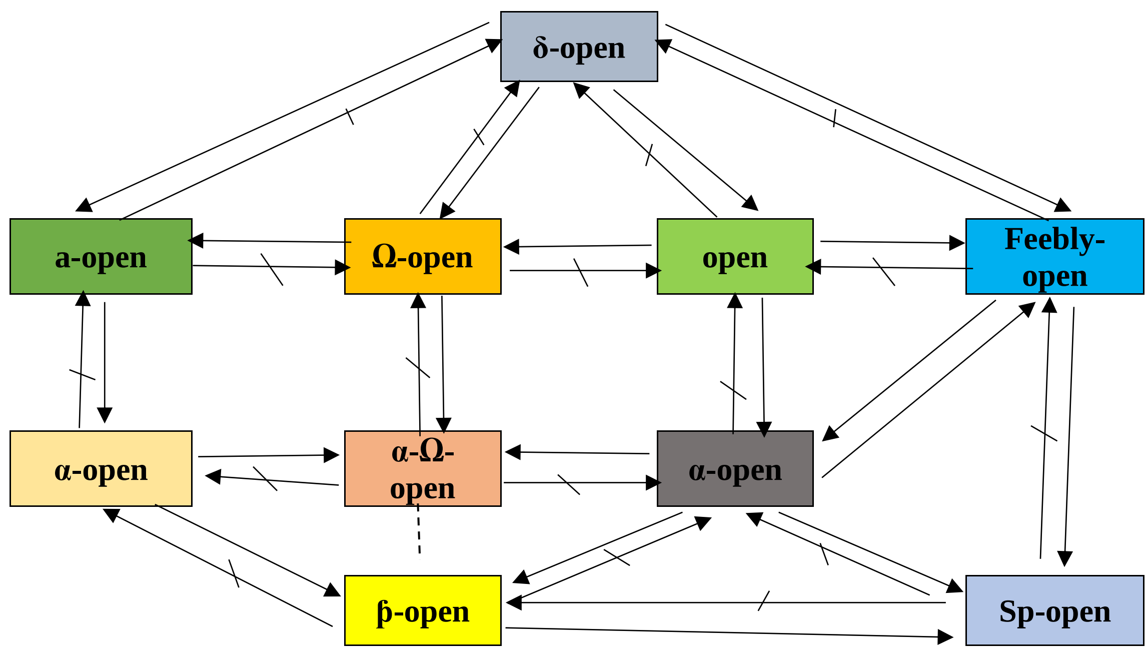

Figure -1-

3. Results and Discussion

We reached new relations between the fuzzy δ-open set and the fuzzy opens set that we studied , And we gave examples to prove that .

Reference :

[1] A.S.Bin shahana , " On Fuzzy strong semi continuity and fuzzy pre continuity , fuzzy set and systems " , 44 (1991) 303-308

[2] A.M.Zahran , " Fuzzy δ-continuous , fuzzy almost regularity on fuzzy topology no fuzzy set " ,fuzzy mathematics ,Vol.3 ,No.1 , 1995 , pp.89-96

[3] Bai Shi – Zhong , Wang Wan – Liang " Fuzzy non – continuous mapping and fuzzy pre – semi – separation axioms " Fuzzy sets and systems Vol.94 pp.261 – 268(1998).

[4] Chang. C.L " Fuzzy topological spaces" J.Math. Anal. Appl.,24,pp.182-190(1968).

[5] Dontchev J. and Przemski M. , " On the various decomposition of continuous and some weakly continuous functions " , Acta mathematica hungarica , vol.71 , no.1-2, pp.109-120 , 1996

[6] H.Z.Hdeib , ''Ω-closed mapping'' , Revista Colomb. De Matem., 16(1-2)(1982), 65-78

[7] Kandil1 A. , S. Saleh2 and M.M Yakout3 "Fuzzy Topology On Fuzzy Sets: Regularity and Separation Axioms" American Academic & Scholarly Research Journal Vol. 4, No. 2, March (2012).

[8] Maheshwari S.N. and Jain P.G. , " Some new mappings " , mathematica , vol.24(47)(1-2)(1982) , 53-55

[9] Mahmoud F. S, M. A. Fath Alla, and S. M. Abd Ellah, "Fuzzy topology on fuzzy sets: fuzzy semicontinuity and fuzzy semiseparation axioms," Applied Mathematics and Computation, vol. 153, no. 1, pp. 127–140, (2003).

[10] Mashhour A.S. and and others "On Pre-continuous and weak Pre-continuous mapping " Proc.Math and Phys.Soc.Egypt 53(1982) , 47-53

[11] Mashhour A.S, Ghanim M.H.and Fath Alla M.A."α-separation axioms and α-compactness in fuzzy topological spaces" Rocky Mountainy J.math, Vol.16, pp.591-600, (1986).

[12] Ming, P. P. and Ming, L. Y., "Fuzzy Topology I. Neighborhood Structure of a Fuzzy point and Moor-smith Convergence", J. Math. Anal. Appl., Vol.76, PP. 571-599, 1980.

[13] Otchana and others , "Ω-open sets and decompositions of continuity " , bulletin of the international mathematical virtual institute , vol.6(2016) , 143-155

[14] Shadman R. Karem "On fuzzy β- separation axioms in fuzzy Topological Space on fuzzy sets"M.SC ,Thesis

,college of science, Koya university,(2008).

[15] Sinha , S . P.,"separation axioms in fuzzy topological spaces", fuzzy sets and system,NO 45,PP.261- 270(1992).

[16] Wong, C. K., "Fuzzy Points and Local Properties of Fuzzy Topology", J. Math. Anal. Appl., Vol.46, PP. 316-328, 1973.

[17] Zadeh, L.A., "Fuzzy Sets", Inform. Control, Vol.8, PP. 338-353, 1965.