Analytical Model of Pulse Amplification in Erbium Doped Fiber Amplifier

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Abstract

Doping a part of the optical fiber core by (Er^{3+}) ions in presence of external pumping power will lead to form the erbium-doped fiber amplifier (EDFA).The performance of this optical amplifier depends on (the power and the wavelength of the pumping laser, the power and wavelength of the input signal, amplifier length, ion concentration). These parameters will affect the characteristics of EDFA such as amplifier gain, gain saturation, noise figure and output power. However, these characteristics can be determined by solving the EDFA propagation and rate equations. The solution of these equations of two-level laser medium can be done numerically. In this paper, we are proposed a novel method to solve these equations. The reconstructed results are perfectly coincided the well known numerical results.

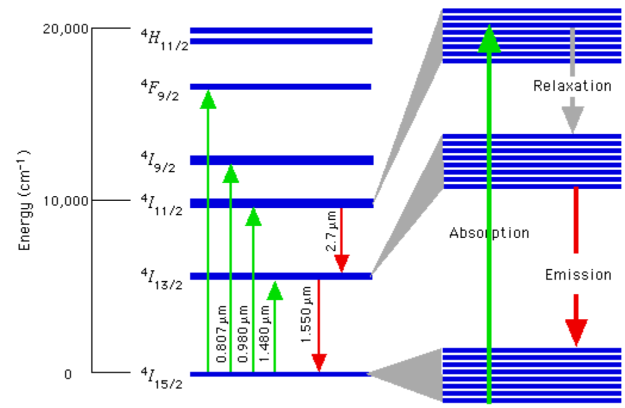
Keywords: EDFA, single mode fiber, population inversion.

1. Introduction

In 2012, H. Sariri, et al [1] studied the effect of ASE on the gain modulation in EDFAs. The derivation of an analytical model for EDFA over modulation response has been presented.

In 2013, P. Sharma and et al [2] discussed various gain flattening techniques. EDFA is widely used amplifier due to its transient suppression, wideband variable gain operation. Electronic feedback control is designed for achieving flat gain at the top of required gain range.

EDFA is fiber amplifier which use the rare-earth elements as a gain medium by doping the fiber core with erbium ions. Although doped-fiber amplifiers were studied as early as 1964, their use became practical only 25 years later [3]. The erbium-doped has made a tremendous progress. It was created a revolution in long distance optical communication systems. EDFA made by doping the silica fiber with erbium ions can operate in a broad range within the 1550 nm window at which the attenuation of silica fiber is minimum and therefore it is ideal for the optical fiber communication systems operating at this wavelength range. These active fibers are finding diverse applications in optical amplifiers, lasers, switches, and a variety of nonlinear devices [4]. The amplifier is modeled as a three level system having three populations of erbium atoms are of interest here: i) the ground state with population density N1; ii) metastable level with population density N2, and iii) pump level with population density N3. In practice, transition from level 3 to level 2 are much more likely than transitions back to the ground state (level 3 to level 1) or the rate of spontaneous emission from state 2. Under



this assumption, N3≈0 [5] .

Fig.(1): Energy level diagram of erbium ions [6].

Here the three-level system has been reduced to an effective two-level system [7]. Population inversion is achieved by injecting power into the system through an external energy source, which is known as pumping. Pumping will excite atoms into the upper energy level 2 [8,9]. Under the assumption that the pump wavelength is in the 1480 nm region. The EDFA can be modeled using the propagation and rate equations for a homogeneous two-level laser medium [10]. The stimulated absorption and emission rates between the ground state and the metastable state are denoted by W_{12} and W_{21}, respectively. Stimulated absorption rate W_{12}, stimulated emission rate W_{21} and transition rates such as pump rate R_{12} are defined as [11]

$$\boldsymbol{W_{12} = \Gamma_s \frac{\sigma_{as}}{h\upsilon_s A_{eff}}\left[P_s + P_{ASE}^{+} + P_{ASE}^{-}\right]} \quad \textbf{(1)}$$

$$\mathbf{W_{21}} = \boldsymbol{\Gamma_s \frac{\sigma_{es}}{h\upsilon_s A_{eff}}\left[P_s + P_{ASE}^{+} + P_{ASE}^{-}\right]} \quad \textbf{(2)}$$

$$\mathbf{R_{12}} = \boldsymbol{\Gamma_p \frac{\sigma_{ap}}{h\upsilon_p A_{eff}} P_p} \quad \textbf{(3)}$$

where σ_{se}, σ_{sa} are the emission and absorption cross sections at signal frequency $\upsilon_s = c/\lambda_s$, and σ_{pa} is the absorption cross section at pump frequency $\upsilon_p = c/\lambda_p$, h is the Plank's constant, c is the speed of light in vacuum, A_{eff} is the effective cross-sectional area of the distribution of erbium ions, P_s and P_p are the signal and pump powers, P_{ASE}^{+} and P_{ASE}^{-} are the forward and backward spontaneous emission power, and Γ_s and Γ_p are the confinement factors of the signal and pump, which represents the overlapping of the optical mode and the erbium-ion distribution. Only the portion of the optical mode which overlaps with the erbium ion distribution will stimulate absorption or emission from the Er^{+3} transitions. Since the erbium is confined to the core of the optical fiber, the mode intensity can more readily invert the erbium ions [5,11]. Spontaneous decay rate A_{21} depends on the fluorescence lifetime (τ_{spon}) of the excited energy level, hence, it is defined as $A_{21} = 1/\tau_{spon}$ [12] where $\boldsymbol{\tau_{spon} = 10m\,\mathbf{sec}}$. The population densities of the two states, N_1 *and* N_2, satisfy the following two rate equations [9]

$$\boldsymbol{\frac{dN_1}{dt} = -W_{12}N_1 + W_{21}N_2 - R_{12}N_1 + A_{21}N_2} \quad \textbf{(4)}$$

$$\mathbf{\frac{dN_2}{dt}} = \boldsymbol{W_{12}N_1 - W_{21}N_2 + R_{12}N_1 - A_{21}N_2} \quad \textbf{(5)}$$

where N_1 is the population density of the ground state, N_2 is the population density of the metastable level. The terms in last equations represent: $W_{12}N_1 =$Number of absorption from level 1 to level 2 due to the signal at υ_s, $W_{21}N_2 =$number of stimulated emission from level 2 to level 1 due to the signal at υ_s, $R_{12}N_1 =$number of absorption from level 1 to level 2 due to the pump at υ_p, $A_{21}N_2 =$number of spontaneous emission from the level 2 to level 1. The total erbium-ion density per unit volume is defined as $N_t = N_1 + N_2$. Steady-state analysis is applicable when the rate of change of variable is much slower than the inherent time constants of the system. Mathematically, the steady-state solutions are obtained by setting the time derivatives in turn, the population densities will be [13]

$$N_1 = N_t \frac{1 + W_{21}\tau_{spon}}{1 + W_{12}\tau_{spon} + W_{21}\tau_{spon} + R_{12}\tau_{spon}} \quad (6)$$

$$N_2 = N_t \frac{R_{12}\tau_{spon} + W_{12}\tau_{spon}}{1 + W_{12}\tau_{spon} + W_{21}\tau_{spon} + R_{12}\tau_{spon}} \quad (7)$$

2. Propagation Equations

The equations that describe the propagation of P_s, P_p, P_{ASE}^+, and P_{ASE}^- are written as bases on the Giles and Desurvire model [14]

$$\frac{dP_s^+}{dz} = P_s^+\Gamma_s\left(\sigma_{se}N_2 - \sigma_{sa}N_1\right) + \alpha_s P_s^+ \quad (8)$$

$$\frac{dP_p^\pm}{dz} = P_p^\pm\Gamma_p\left(\sigma_{pe}N_2 - \sigma_{pa}N_1\right) - \alpha_p P_p^\pm \quad (9)$$

$$\frac{dP_{ASE}^\pm}{dz} = \pm P_{ASE}^\pm\Gamma_s\left(\sigma_{se}N_2 - \sigma_{sa}N_1\right) \pm 2\sigma_{se}N_2\Gamma_s h\upsilon_s\Delta v \mp \alpha_s P_{ASE}^\pm \quad (10)$$

where z is the coordinate along the EDFA, The signal positive (+) means a forward propagation beam and signal negative (-) for backward propagation, $P_s^\pm$ is the forward and backward signal power, respectively. P_p^+ is the pump power of the EDFA in forward direction, $P_{ASE}^\pm$ are the forward and backward amplified spontaneous emission (ASE) of the EDFA, respectively. The second term on the right hand side of Eq.(10) is the spontaneous noise power ($P_{ASE}^o = h\upsilon_s\Delta v$) of the EDFA within the EDFA homogeneous bandwidth (Δv) [15]. This bandwidth noise can be estimated from $\Delta v = \int_0^\infty \left(\sigma_e(v)/\sigma_{e,peak}\right)dv$. Thus the two-level amplifier system can be fully characterized using Eqs.(6) to (10) that describe the propagation of the signal, pump and ASE along the EDFA. Eqs.(8) to (10) can be solved analytically to extracted the total amplifier gain G for an EDFA of length L. In this analysis, the signal and ASE terms are assumed to have both forwards and backwards propagating components [16]. α_s and α_p are the absorption coefficients per unit length of signal and pump, respectively, which are defined as [14] $\alpha_s = \Gamma_s\sigma_{sa}N_t$ and $\alpha_p = \Gamma_p\sigma_{pa}N_t$. These definitions govern the evolution of signal and pump powers inside an EDFA. In short fibers, these losses are negligible. However, they should be taken into account for long fibers specifically distributed erbium doped fiber.

3. Present Analytical Model

There are many studies published in the scientific papers [8] analyzed the performance of EDFA according to Eqs.(8) to (10), where these equations were solved numerically. However, we propose a new analytical solution. We see that the behavior of the erbium ion concentration of level 1 and 2 is constant along most of the length of the amplifier (and that is depends on the pump power) after that the concentration decreases (for N_2) or increases (for N_1) smoothly. So, we can assume that for short lengths, the values of N_2 and N_1 are constants. So our model assumes that the amplifier consists of many equal short concatenate segments. We assumed that the concentrations are constant along z for each segment of EDFA. So these equations way be solved analytically. Our assumption is necessary to consider the differential equations with constant coefficient. To clarify the derivations, let

$$A = \Gamma_p \left(\sigma_{pe} N_2 - \sigma_{pa} N_1 \right) - \alpha_p \quad (11)$$

$$B = \Gamma_s \left(\sigma_{se} N_2 - \sigma_{sa} N_1 \right) - \alpha_s \quad (12)$$

$$C = 2\sigma_{se} N_2 \Gamma_s h \upsilon_s \Delta v \quad (13)$$

Substituting $A, B,$ and C into Eq.(8) to (10), yields

$$\frac{dP_p^{\pm}}{dz} = \pm A P_p^{\pm} \quad (14)$$

$$\frac{dP_s^{+}}{dz} = B P_s^{+} \quad (15)$$

$$\frac{dP_{ASE}^{\pm}}{dz} = \pm B P_{ASE}^{\pm} \pm C \quad (16)$$

Note that, according to our assumptions, the parameter $A, B,$ and C will be invariant through each segment. Their variations are limited in the end points of the segments only. The solution of the forward pump equation in the concatenated sections will be

$$P_p^{+}(\Delta z) = P_p^{+}(0) e^{A_1 \Delta z}$$

$$\vdots$$

$$P_p^{+}(N\Delta z) = P_p^{+}\left((N-1)\Delta z\right) e^{A_N \Delta z}$$

These equations may be generalized to yield

$$P_p^{+}(L) = P_p^{+}\left((N-1)\Delta z\right) e^{A_N \Delta z} \quad (17)$$

$L = N.\Delta z$ represents the length, where N is the number of segments and equals 601, $P_p^{+}(0)$ is the initial pump power that inters in the forward direction. It is important to note that the coefficients $A's$ may be positive or negative depending on the operating conditions. The entire description of variation depends on the recursive solution in the concatenated segments. The recursion formula can be deduced as

$$P_p^{+}(i\Delta z) = P_p^{+}\left((i-1)\Delta z\right) e^{A_i \Delta z} \quad (18)$$

where i is an integer number. The solution of the backward pump equation for the concatenated segments will be

$$P_p^{-}(L - N\Delta z) = P_p^{-}\left(L - (N-1)\Delta z\right) e^{A_N \Delta z}$$

The following recursion formula can be deduced

$$P_p^{-}(L - i\Delta z) = P_p^{-}\left(L - (i-1)\Delta z\right) e^{A_{N-i+1} \Delta z} \quad (19)$$

$P_p^{-}(0)$ is the initial pump power that inters in the backward direction. Using a similar procedure to signal propagation equations, we get the recursion formula

$$P_S^{+}(i\Delta z) = P_S^{+}\left((i-1)\Delta z\right) e^{B_i \Delta z} \quad (20)$$

where $P_s^{+}(0)$ is the initial signal power that inputs in the forward direction.

The propagation equations that describe P_{ASE}^{+} and P_{ASE}^{-} are non-homogeneous differential equations and can be solved as following : First, for forward ASE, we have the solution

$$P_{ASE}^{+}(z) = ke^{Bz} - \frac{C}{B} \tag{21}$$

where the constant k may be determined using the boundary conditions at $z=0$ to yield

$$k = P_{ASE}^{+}(0) + \frac{C}{B} \tag{22}$$

Substituting Eq.(22) into (21), yields

$$P_{ASE}^{+}(z) = P_{ASE}^{+}(0)e^{Bz} + \frac{C}{B}\left(e^{Bz} - 1\right) \tag{23}$$

The last equation may be reformed for the concatenated segments as

$$P_{ASE}^{+}(N\Delta z) = P_{ASE}^{+}\left((N-1)\Delta z\right)e^{B_N\Delta z} + \frac{C_N}{B_N}\left(e^{B_N\Delta z} - 1\right)$$

From these equations one can deduce the recursion formula

$$P_{ASE}^{+}(i\Delta z) = P_{ASE}^{+}\left((i-1)\Delta z\right)e^{B_i\Delta z} + \frac{C_i}{B_i}\left(e^{B_i\Delta z} - 1\right) \tag{24}$$

where $P_{ASE}^{+}(0)$ refers to the initial spontaneous emission power for the forward direction. Second, for backward ASE, using a similar steps for forward ASE, the reconstructed P_{ASE}^{-} will be

$$P_{ASE}^{-}(0) = P_{ASE}^{-}(\Delta z)e^{-B_1\Delta Z} + \frac{C_1}{B_1}(e^{B_1\Delta z} - 1)$$

In this case, the resulted recursion from is

$$P_{ASE}^{-}(L - i\Delta z) = P_{ASE}^{-}(L-(i-1)\Delta z)e^{-B_{N-i+1}\Delta Z} + \frac{C_{N-i+1}}{B_{N-i+1}}(e^{B_{N-i+1}\Delta z} - 1) \tag{25}$$

The initial value of $P_{ASE}^{+}(0)$ in forward direction is zero and the initial value of $P_{ASE}^{-}(L)$ in the backward direction is zero.

4. Calculation of P_{ASE}^{-}

There is no problem in calculating P_{ASE}^{+} because the initial condition is known in z=0, so we can solve Eq.(23) numerically or by the proposed method beginning from z=0 for the first segment .The calculated P_{ASE}^{+} of the first segment becomes the initial value for the second segment and so on. This is correct in using numerical methods or our proposed method. But when we start to calculate P_{ASE}^{-} in the first segment , we found that there is no known initial value in z=0.,so, the solution of Eq.(25) for P_{ASE}^{-} must begin from the end of the fiber (z=L) because the initial value $P_{ASE}^{-}(L) = 0$. And by solving Eq.(25) beginning from the last segment (N) in backward direction with taking the value of $P_{ASE}^{-}(L) = 0$ as initial value we get the value of $P_{ASE}^{-}(L - \Delta z)$ for (N-1) segment . We take $P_{ASE}^{-}(L - \Delta z)$ as the initial value to calculate the value of $P_{ASE}^{-}(L - 2\Delta z)$ for the (N-1) segment ,and so on. We are obliged to calculate Eqs(1-2) and the related equations three times. In the first one, we calculate P_{ASE}^{+} only and P_p^{+} , P_S^{+} for all segments in forward direction . We put $P^{-}_{ASE} = 0$ for all segment. In the second time, we begin the calculation from the last segment of EDFA using the result of the forward calculation to find the values of P^{-}_{ASE} using $P^{-}_{ASE}(L) = 0$ as initial value. In the third time , we repeat the first calculation using the stored values of P^{-}_{ASE} beginning from the first segment in forward direction.

5. Results and Discussion

The numerical method was simulated by the Matlab program. The EDFA is divided into many segments, for example N=601 segments, each of them with length L/N. Using Eqs.(14) to (16) to determine the output signal ,pump and ASE powers. we calculate the solution of Eqs.(14) to (16) using our proposed model .we repeat the above calculation by using 4th order Runge-Kutta method to compare with the new model. we found that the results of P_p , P_S , show that the two methods give approximately (more than 99%) the same results .also, there is a small difference between the values of P^{-}_{ASE} which calculated by the two method , but the difference between them decrease and the two methods give approximately (more than 99%) the same results when we choose small width of the segments. We applied our proposed model in parallel with the numerical method to show the behavior of signal, pump and ASE powers.

Fig (1) shows that the signal powers increase to max value at approximately 6 meter from the beginning of EDFA . After that the value of the signal decreases along the length of EDFA until its value becomes less than its input value. This is because pump power decreases along EDFA length as shown in Fig(2) and that leads to decrease in N2 population as shown in Fig(3) . Fig(4) shows the distribution of ASE power for forward and backward direction .

6. Conclusions

In conclusion, the propagation equations in EDFA was solved. A simple recursive formula is presented to calculate the output power of signal , pump, ASE and. The numerical results show that our formulas agrees well with the numerical solution. We proposed a method to find the solution of the problem of solving the backward ASE propagation equation.

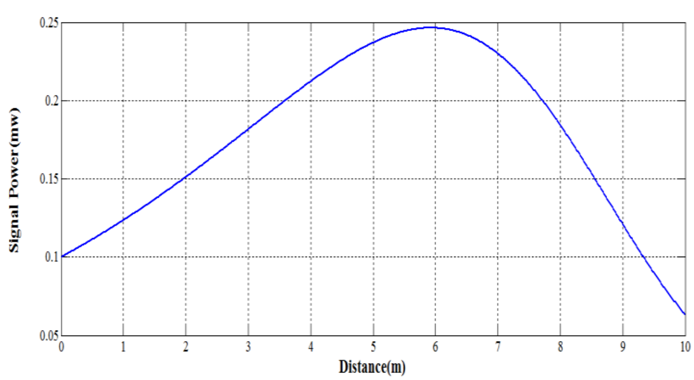


Fig.(2): Signal power Disiribution along EDFA distance.

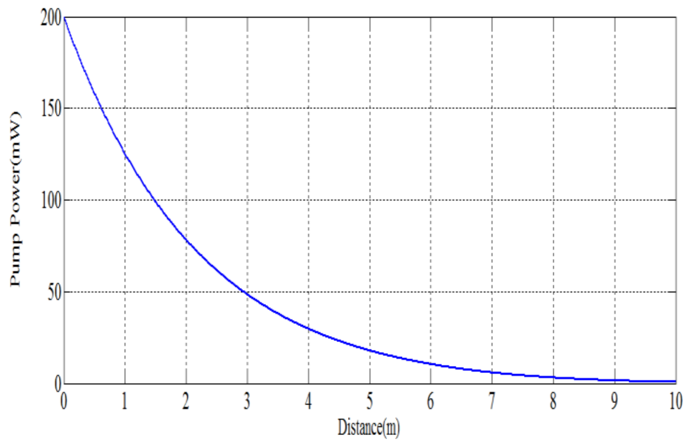


Fig.(3):Disiribution of pump power along EDFA distance

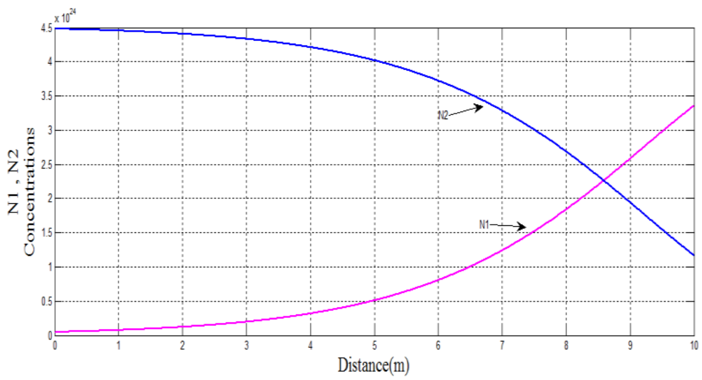


Fig.(4): N1 and N2 populations along EDFA length.

The simple difference between the values of our calculated by numerical method and Runge-Kutta method

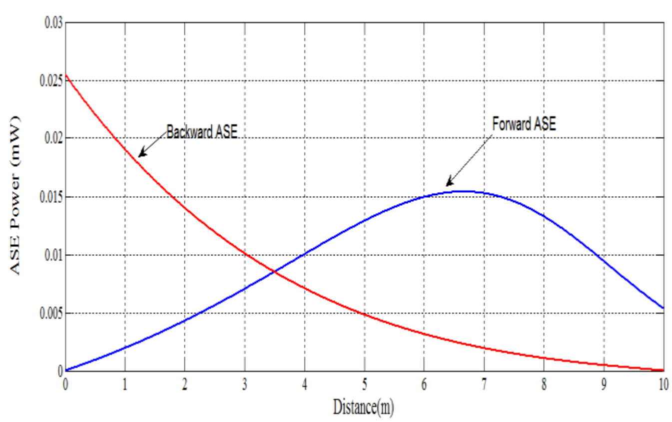


Fig.(4): Power distribution of Forward and backward ASE along EDFA length.

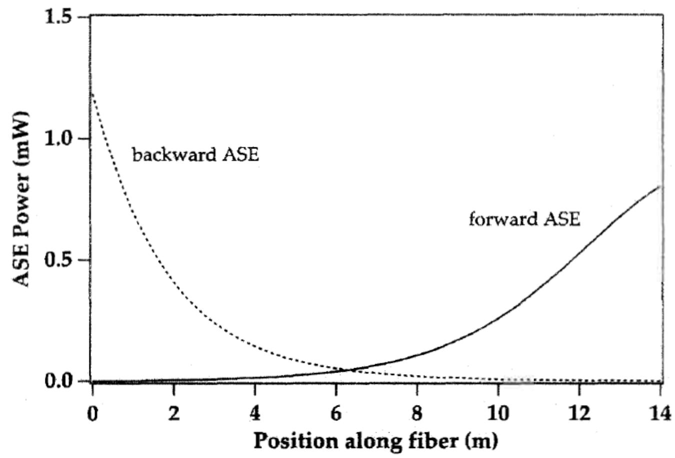


Fig.(5): Power distribution of Forward and backward ASE along EDFA length by Runge-Kutta method [18].

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