# Nuclear Structure of the ($^{144}$Ba, $^{146}$Ce $^{148}$Nd )by using the Interacting boson Approximation (IBA-2)

Zainab Sauod Muhmmed Alhmod

*Physics department ,College of Science, Al Muthanna University*

*Corresponding author: zainablahmod@mu.edu.iq

*Received:24/12/2017, Accepted:21/1/2018, published: 26/03 /2018*

**DOI: 10.52113/2/05.01.2018/20-26**

**Abstract:** The Interacting Boson Hamiltonian (IBA-2) to reproduce energy spectrum of isotones ($^{144}Ba_{56}$, $^{146}Ce_{58}$ *and* $^{148}Nd_{60}$ ) is tested The calculated ratio $E_{4_1}/E_{2_1} \approx 2.5$ has been suggested that these nuclei are part of O(6) group symmetry. Reduced electric quadruple transitions between states of $\Delta I = 0,2\ I \neq 0$ and magnetic dipole between states of $\Delta I = 0,1\ I \neq 0$, are calculated. The mixing ratios $\delta(E2/M1)$ for transitions between $\Delta I = 0,1\ I \neq 0$ are evaluated as well. All the calculated results have been compared with available experimental data and satisfactory results are obtained.

***Keywords:*** Interacting boson model (IBA-2), even – even( $^{144}$Ba,$^{146}$Ce,$^{148}$Nd), quadruple moments, mixing ratios, energy levels

## 1. Introduction

In the last few years there has been a dramatic increase in the available spectroscopic information relating to neutron-rich nuclei, like the nuclei under study in this work, [1-3]. These results represent a challenge for the nuclear physics theorist to develop and modernize their models to understand well the nuclear structure. The Interacting Boson Model (IBA) is one of those attempts that have been successful, for a long time, in describing the low lying nuclear collective motion in different mass regions. The first version of IBA, initially introduced by Arim and I Chellos [4], which has been rather successful in describing the collective properties of several medium and heavy nuclei. Then the version of IBA-2, in which there is distinguish between proton and neutron bosons wave functions [5]. Later there are IBA-3 and IBA-4.[6-8] In the IBA-1 and IBA-2 versions, the bosons consider as a pair of the same kind of nucleons with angular momentum L=0 (called s boson) and L=2 (called d boson). While in the IBA-3 there are a third kind of bosons (called $\pi$ boson), in which there pear are of different kinds of particles. In the IBA-4 we have extra boson with L=4 called (g-bosons). The last two versions are limited to a certain region of nuclei. In the present work, the IBA-2 version will be used to calculate different nuclear properties of the neutron rich isotone

$$^{144}Ba_{56},\ ^{146}Ce_{58}\ and\ ^{148}Nd_{60}$$

## 2-The Interacting Boson Model

The Hamiltonian operator in IBA-2, which has been used in the calculation of the energy levels and hence the gamma transitions matrix elements, has three parts, one for proton bosons, one for neutron bosons and the third one that describes the interaction between unlike bosons:

$H = H_\pi + H_\nu + H_{\pi\nu}$ (1)

The Hamiltonian generally used in phenomenological calculations can be written as $H = \varepsilon_d(n_{d\nu} + n_{d\pi}) + \kappa(\varrho_\nu.\varrho_\pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi}$ (2) calculated and experimental energy levels of $^{146}$Ce where the dot denoted the scalar product. The first term represents the single-boson energies for neutron and protons, $\varepsilon_d$ is the energy difference between s- and d-boson and $n_{d\rho}$ is the number of d-bosons, where $\rho$ correspond to $\pi$ (proton) or $\nu$ (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e. the quadrupole-quadrupole interaction between neutron and proton bosons with the strength $\kappa$. The quadruple operator $\varrho_\rho = \left[d_\rho^+ s_\rho + s_\rho^+ d_\rho\right]^{(2)} + \chi_\rho\left[d_\rho^+ d_\rho\right]^{(2)}$ (3)

Where $\chi_\rho$ determines the structure of the quadruple operator and is determined empirically. The square bracket in Eq. (3) denotes angular momentum coupling. The terms $V_{\pi\pi}$ and $V_{\nu\nu}$, in equation (2) which correspond to interaction between like-boson, are sometimes included in order to improve the fit to experimental energy spectra. They are of the form

$$V_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_L^\rho \left(\left[d_\rho^+ d_\rho^+\right]^{(L)}.\left[d_\rho d_\rho\right]^L\right). \quad (4)$$

However, their effects are usually considered minor and often neglected .

The Majorana term, $M_{\nu\rho}$, which contains three parameters $\xi_1$ ,$\xi_2$ and $\xi_3$ may be written as

$$M_{\nu\pi} = \frac{1}{2}\xi_2\left([S_\nu^+ d_\pi^+ - d_\nu^+ s_\pi^+]^{(2)}.[s_\nu d_\pi - d_\nu s_\pi]^{(2)}\right) - \sum_{k=1,3}\xi_k\left([d_\nu^+ d_\pi^+]^{(k)}.[d_\nu d_\pi]^k\right). \quad (5)$$

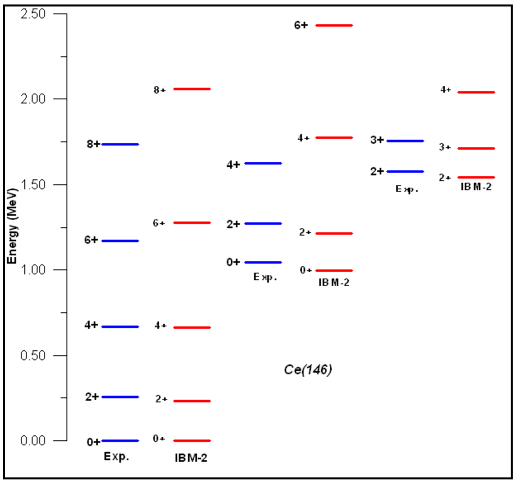


Fig 1 : A comparison between calculated and experimental energy levels of $^{146}$Ce

**3- Calculations and results**

The isotone chosen in this work are A=144,146 and 148 due to the presents of experimental data. The energy spectrum and electromagnetic properties of nuclei in this region have been investigated by several others by using different theoretical calculations [9-12]. We have fixed neutron number N=88, means $N_\nu = 3$ (taken the magic number 82 as a closed shell) and $N_\pi$ varies from 3 in ($^{144}$Ba) to 5 in ($^{148}$Nd), measured from the closed magic shell at 50. The model parameters $\kappa, \chi_\rho, \varepsilon_\rho$ and Majorana parameters $\xi_k$, with k=1,2,3, were treated as free parameters and their values were estimated by fitting with the experimental values of energy levels. The Majarona parameters have greater effect on the energy of $2_3$ , mixed symmetry state, which has been fitted according to the values of these parameters. The procedure was made by selecting the traditional values of the parameters and allowing one parameter to vary while keeping the others fixed until the best fit with the experimental obtained. This was carried out until one overall fit was obtained. The best fit values for the Hamiltonian parameter are given in Table-1. Concentration was made on the $2_1^+$ to make a reasonable fit to experimental data. The fitting results are shown in Table-2. The ratio $E_{4_1}/E_{2_1}$ presented in table-3 with its experimental value which confirms that these isotone are a part of gamma soft O(6) limit of the IBA-1 and lies very closed to one edge of the Casten Triangle, recently called Casten prism [13,14]. A sample of experimental and theoretical scheme is presented in Fig.1. An overall a good agreement was obtained for the ground, beta and gamma band. A detail comparison between experimental and theoretical energy levels one can find that the fit of the ground is very good for the $2_1$ and $4_1$, but the model cannot predicts the $6_1$ correctly, this may be due to the high spin of this state. Actually this has slim effects on calculations of transition probabilities. The fit of the $2_3^+$ in all isotone, which recognized as a mixed symmetry state [9], is very good due to the effect of changing the Majarona parameters and the electromagnetic properties as following:

1- E2 transition and quadruple moments

In order to understand how the IBA-2 Hamiltonian reflects other physical properties of the nuclear system, the wave function obtained from diagonalization of H to calculate the reduced electric quadruple transition probability and the quadruple moment of the state $2^+$.In IBA-2, the E2, transition operator is given by,

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \tag{6}$$

where $Q_\rho$ is the same as in equation (3) and $e_\pi$ *and* $e_\nu$ are boson effective charges depending on the boson number $N_\rho$ and they can take any value to fit the experimental results. The method of estimation the effective charges value in explained in reference [16]. The effective charges calculated by this method for the three isotone are presented table-4.The results of calculations are shown is Table-5. As one can see from the table, we have quit good agreement between experimental data and the calculated one. We have some discrepancies in some values. The quadruple moments of $2_1^+$ were calculated and the values listed in Table 6. Only one piece of experimental data is available. From the experimental value of $(B(E2);2_1-0_1)$ one can calculate the quadruple moment, using the relation [17]

$$B(E2;0\rightarrow 2) = \frac{5}{16\pi}Q_0^2$$

$$Q(2_1^+) = -\tfrac{2}{7}Q_0 \tag{7}$$

where $Q_o$ is the classical quadruple moment.

Although the relations are for the collective deformed nuclei, but the large value of quadruple moment inviting uses them. The predicted values always greater than one and they couch the negative sign of the experimental value.

2- The M1 transition and $\delta(E2/M1)$ mixing ratios: The M1 operator obtained by supposing $l=1$ in the single boson operator of the IBA-2 and can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{\frac{1}{2}} \left(g_\pi L_\pi{}^{(1)} + g_\nu L_\nu{}^{(1)}\right) \quad (8)$$

where $g_\pi, g_\nu$ are the boson g-factors in units of $\mu_N$ and $L^{(1)} = \sqrt{10}(d^+ x\tilde{d})^{(1)}$. This operator can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{\frac{1}{2}} \left[\frac{1}{2}(g_\pi + g_\nu)\left(L_\pi{}^{(1)} + L_\nu{}^{(1)}\right) + \frac{1}{2}(g_\pi - g_\nu)\left(L_\pi{}^{(1)} - L_\nu{}^{(1)}\right)\right] \quad (9)$$

The first term on the right hand side ,in the above equation, is diagonal and therefore for M1 transitions the previous equation may be written as

$$T^{(M1)} = 0.77\left[(d^+ d^\sim)_\pi^{(1)}(g_\pi - g_\nu)\right] \quad (10)$$

The direct measurement of B(M1) matrix elements is difficult normally, so the M1 strength of gamma transition may be expressed in terms of the multipole mixing ratio which can be written as [16]

$$\delta\left({E_2}/{M_1}\right) = 0.835 E_\gamma(MeV).\Delta \quad (11)$$

Where $\Delta = \frac{\langle I_f \| T^{E_2} \| I_i \rangle}{\langle I_f \| T^{M_1} \| I_i \rangle}\ in\ eb/\mu_N$

Having fitted E2 matrix elements, one can then use them to obtain M1 matrix elements and then the mixing ratio $\delta({E2}/{M1})$ . The $g_\pi\ and\ g_\nu$ in equation (10), have to be estimated, if they have not had been measured. The g factors might estimate from experimental magnetic moment $(\mu)$ of the $2^+{}_1$ state $\mu = 2g_{total}$ and $\mu = 0.64(8)$. The total gyro magnetic ratio wrote by Sambataro et al [18] as;

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \quad (12)$$

Many relations could be obtained for a certain mass region and then the average $g_\pi\ and\ g_\pi$ values for this region could be calculated. One of the experimental B(M1) and the relation above used to find that $g_\pi - g_\nu = 0.017\mu_N$ . one has to remember that the traditional values of the g factor should obey the relation $g_\rho + g_\nu = 1$.The estimated values of the parameter are $g_\pi = 0.33\mu_N\ and\ g_\nu = 0.31\mu_N$ ,these were used to calculate the ratio $\Delta(E2 \backslash M1)$ and then the mixing ratio $\delta(E2 \backslash M1)$. The ratios were calculated for some selected transitions in ${}^{148}Nd_{60}$ due to existence of experimental data to compare with, the result listed in Table-7.

## 23. Conclusion

In summary, the nuclear structure of the even-even, N=88, isotone is well presented in this work by using program codes (NPBOS and NPTRN) of the IMB-2. Many nuclear properties of these nuclei studied. This work increased the knowledge of thin region. The almost fixed ratios of (E4/E2) give indication that these isomers are very closed to the gamma-soft O(6), nuclei. The value of neutron effective charges are always greater than the effective charges of proton, which is up normal, but one can explain this by regarding the unit of the effective charge which is $fm^2$ (unit of area). Means the neutrons occupy greater area in the nucleus than proton, which is real. The sign of the multiple mixing ratios is produced well by the model.

Table- 1: The IBA-2 parameters

| **Nucleus** | $N_\pi$ | $N_\nu$ | **N** | $\varepsilon$ | $\kappa$ | $\chi_\pi$ | $\chi_\nu$ | $C_{0,2,4}$ | $\xi_1=\xi_3,\xi_2$ |
|---|---|---|---|---|---|---|---|---|---|
| $^{144}_{56}Ba$ | 3 | 3 | 6 | 0.500 | -0.142 | -1.2 | -0.62 | 0.16,0.16,0.0 | -0.015,0.099 |
| $^{146}_{58}Ce$ | 4 | 3 | 7 | 0.660 | -0.142 | -1.2 | -0.62 | 0.15,0.15,0.0 | 0.04,-0.040 |
| $^{148}_{60}Nd$ | 5 | 3 | 8 | 0.848 | -0.142 | -1.2 | -0.62 | 0.18,0.18,0.0 | -0.080,0.045 |

Table-2: The calculated energy level of IBA-2 compared with available experimental data from reference [15]

| **Nucleus** | **calculations** | **$2_1$** | **$4_1$** | **$6_1$** | **$8_1$** | **$0_2$** | **$2_2$** | **$3_1$** | **$2_3$** |
|---|---|---|---|---|---|---|---|---|---|
| $^{144}Ba_{56}$ | Exp. = | **0.199** | **0.530** | **0.961** | **1.470** | **1.020** | **1.315** | | **1.560** |
| | IBA-2= | 0.199 | 0.588 | 1.151 | 1.878 | 0.988 | 1.139 | 1.605 | 1.490 |
| $^{146}Ce_{58}$ | Exp. = | **0.258** | **0.668** | **1.170** | **1.736** | **1.043** | **1.272** | **1.577** | **1756** |
| | IBA-2= | 0.237 | 0.665 | 1.279 | 2.007 | 0.996 | 1.212 | 1.713 | 1.544 |
| $^{148}Nd_{60}$ | Exp. = | **0.302** | **0.752** | **1.280** | **1.856** | **0.916** | **1.170** | **1.511** | **1.249** |
| | IBA-2= | 0.301 | 0.783 | 1.432 | 2.180 | 1.000 | 0.907 | 1.208 | 1.216 |

Table-3: The ratio E4/E2 and E6/E2 with the experimental values

| **Nucleus** | **calculations** | **E4/E2** | **E6/E2** |
|---|---|---|---|
| $^{144}Ba_{56}$ | Exp. = | **2.66** | **4.82** |
| | IBA-2= | 2.95 | 5.79 |
| $^{146}Ce_{58}$ | Exp. = | **2.59** | **5.04** |
| | IBA-2= | 2.87 | 5.5 |
| $^{148}Nd_{60}$ | Exp. = | **2.49** | **4.25** |
| | IBA-2= | 2.60 | 4.76 |

Table-4: The IBA-2 effective charges

| **Isotones** | $e_\pi\ efm^2$ | $e_\nu\ efm^2$ |
|---|---|---|
| $^{144}Ba_{56}$ | 0.041 | 0.215 |
| $^{146}Ce_{58}$ | 0.158 | 0.355 |
| $^{148}Nd_{60}$ | 0.100 | 0.566 |

Table-5: Theoretical and the available experimental values of B(E2) in $e^2b^2$

| Transition | $^{144}Ba$ | | $^{146}Ce$ | | $^{148}Nd$ | |
|---|---|---|---|---|---|---|
| | Exp. | IBA-2 | Exp. | IBA-2 | Exp. | IBA-2 |
| $2_1 \to 0_1$ | 0.208(6) | 0.203 | 0.93(13) | 0.94 | 1.37(2) | 1.431 |
| $2_2 \to 0_1$ | | 0.007 | | 0.025 | 0.075(5) | 0.133 |
| $2_2 \to 2_1$ | | 0.037 | | 0.165 | 0.085(5) | 0.022 |
| $2_3 \to 0_1$ | | 0.002 | | 0.002 | 0.073(3) | 0.025 |
| $2_3 \to 2_1$ | | 0.001 | | 0.006 | 0.026(1) | 0.022 |
| $4_1 \to 2_1$ | 0.407(61) | 0.501 | | 2.628 | 1.44(6) | 2.074 |
| $3_1 \to 2_1$ | | 0.0161 | | 0.047 | | 0.034 |
| $3_1 \to 1_1$ | | 0.0196 | | 0.000 | | 0.602 |
| $0_2 \to 2_1$ | | 0.017 | | 1.288 | 0.005(1) | 0.023 |

Table-6: The values of electric qaudrupole moments of the isotons N=88

| **Isotons** | $[Q\ fm^2]_{\exp.}$ | $[Q\ fm^2]_{theor.}$ |
|---|---|---|
| $^{144}Ba_{56}$ | -0.68(2) | -1.78 |
| $^{146}Ce_{58}$ | -1.37(19) | -1.54 |
| $^{148}Nd_{60}$ | -1.46(13) | -2.18 |

Table-7: The experimental and the calculated mixing ratio for $^{148}Nd_{60}$

| **Isotons** | Transition energy (MeV) | $I_f \to I_i$ | $[\delta(E2/M1)]_{\exp.}$ | $[\delta(E2/M1)]_{theor.}$ |
|---|---|---|---|---|
| $^{148}Nd_{60}$ | 0.869 | $2_2 \to 2_1$ | +8(+12-2) | +6.92 |
| | 0.947 | $2_3 \to 2_1$ | - | -0.34 |
| | 1.209 | $3_1 \to 2_1$ | 0.20(4) | -5.12 |
| | 0.759 | $3_1 \to 4_1$ | +5(+15-22) | -12.1 |
| | 0.976 | $3_2 \to 4_1$ | 0.0(+13-1) | -0.25 |
| | 1.427 | $3_2 \to 2_1$ | +0.37(5) | +60 |

## 5. References

[1] A. Gandea, E. Sahin, J.J Valiente et al, Acta physica Polonica B, Vol. 38, No 4,1311-1319 (2007).

[2] D. Bucurescue, C. Rusa, N. Ma'rginean et al, Phys. Rev. C76, No 6, 064301 (2004).

[3] A. Gade, P. Adrich, D. Bezin et al, Pysics Rev. Lett. 102(18) (2009).

[4] -F. Iachello and A. Arima, Ann. Phys, NY, 111, 201 (1978).

[5] W.Pfeifer "An Interacting Boson IBA), vdf hochschulverlag- Zurich. (1998).

[6] J.P. Elliot, J.A. Evans, V.Lac and G.L. Long, Nuclear physics A 609, 1-20, (1996).

[7] Falih H. Al-Khudair, Chines Physics C Vol. 33, No 7, pp. 538-546,

(2009).

[8] P. Van Isacker, A.Bouldjedri and S.Zerguine, Nucl. Phys. A836, (2010).

[9] W.D. Hamilton, A. Irback and J.P. Elliot, Physics latters, vol. 53, No. 26, p 2469- 2472, (1984).

[10] D.S. Chuu, c.S.Han, S.T.Hsieh and M.M. King Yen, Physical Review C, Vol. 30, no. 4, p 1300-1309, (1984).

[11] S.J.Zhu, Q.H. Lu, J.H. Hamilton et al, Physics letter B, 357, 273-280, (1995)

[12] R. Toams, J. L. Egido, Physics letter B, 663,lssues 1-2,p 49-54, (2008)

[13] R.F. Casten, Romanian Report in Physics, Vol. 57, No. 4, p515-526, (2005)

[14] P.Van Isacker, A. Bouldjedri and S. Zerguine, Nuclear Physics, A836, 225-241(2010).

[15]ENSDF,http://www.nndc.bnl.gov/ensdf (Nuclear data sheet)

[16] A.R.H. Subber, S.j.Robinson, W.D.Hamilton etal, J.Phys. G: nuclear physics.13, 807-837, (1987).

[17] [16]- S.J.Q. Robinson, A. Escuderos, L.Zamick et al, Phys. Rev.

C73, 037306,(2006).

[18] M. Smabatora, O. Scholton, A Deiprink, Nucl. phy. A423 333, (1984).