Analysis of Undamped Oscillator Subjected to Triangular Pulse via Gupta Integral Transform

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ABSTRAC: An undamped (mechanical as well as electrical) oscillator subjected to a triangular pulse is generally analyzed via ordinary methods such as calculus. In this paper, an undamped (mechanical as well as electrical) oscillator subjected to a triangular pulse was analyzed to obtain its response via the Gupta integral transform (GT). It put forward a new technique for obtaining the response of an undamped oscillator subjected to a triangular pulse force and proves that the GT is an effective integral transform than calculus.

Keywords: Gupta integral Transform, undamped oscillator, triangular pulse.

1. Introduction

The load F_o is instantly applied to the structure and decreased linearly over time duration t_1 as shown in the figure 1.

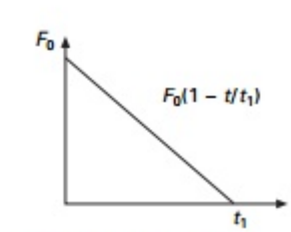

Figure 1- Variation of load with time

The triangular pulse force [1] is written as:

$$F(t) = F_o\left(1 - \frac{t}{t_1}\right) for\ t < t_1$$

$$= 0\ for\ t \geq t_1.$$

The GT is a new integral transform that has been proposed by the authors Rahul Gupta and Rohit Gupta in recent years. It has been applied to solve many initial value problems [2]. In this paper, an undamped (mechanical as well as electrical) oscillator subjected to a triangular pulse was analyzed to obtain its response via the Gupta integral transform (GT). It proves that the GT is an effective integral transform than calculus.

The GT is defined for a function of exponential order as follows:

Considering functions in the set C defined as:

$C = \{g(t): \exists\ R, q_1, q_2 > 0,\ |g(t)| < Re^{q_i|(t)|}, if\ t \in (-1)^i X[0,\infty)\}$.

For a given function in set C, the constant R must be a finite number, q_1 *and* q_2, may be finite or infinite.

The GT of a function $g(t)$ is defined [3], [4] by the integral equations as

$\dot{R}\{g(t)\} = G(q) = \frac{1}{q^3}\int_0^\infty e^{-qt}\, g(t)dt,\ t \geq 0,\ q_1 \leq q \leq q_2$. The variable q in this transform is used to factor the variable t in the argument of the function g.

1.1. GT (Gupta Transform) of Basic Functions

- ❖ $\dot{R}\{t^n\} = \frac{n!}{q^{n+4}}$, $where\ n = 0,1,2,3 \ldots \ldots \ldots$
- ❖ $\dot{R}\{sinat\} = \frac{a}{q^3(q^2+a^2)}$, $q > 0$

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- $\dot{R}\{cosat\} = \frac{1}{q^2(q^2+a^2)}$, $q > 0$

1.2 GT of Unit step function

A unit step function is defined as $U(t-a) = 0\ for\ t < a\ and\ 1\ for\ t \geq$ a.

The GT of unit step function is given by

$$\dot{R}\{U(t-a)\} = \frac{1}{q^3}\int_0^\infty e^{-qt}\,U(t-a)dt$$

$$\dot{R}\{U(t-a)\} = \frac{1}{q^3}\int_a^\infty e^{-qt}\,dt$$

$$\dot{R}\{U(t-a)\} = \frac{1}{q^4}e^{-qa}$$

1.3 Shifting property of Gupta transform

If $\dot{R}\{g(t)\} = G(q),\ then$

$\dot{R}[g(t-d)U(t-d) = e^{-qd}G(q)$.

Proof:

$$\dot{R}[g(t-d)U(t-d) = \frac{1}{q^3}\int_0^\infty e^{-qt}\,g(t-d)U(t-d)dt$$

$$= \frac{1}{q^3}\int_d^\infty e^{-qt}\,g(t-d)dt$$

$$= \frac{1}{q^3}\int_0^\infty e^{-q(k+d)}\,g(k)dk,$$

$$where\ k = t-d$$

$$= e^{-q(a)}\frac{1}{q^3}\int_0^\infty e^{-qk}\,g(k)dk$$

$$= e^{-q(d)}\frac{1}{q^3}\int_0^\infty e^{-q(t)}\,g(t)dt$$

$$= e^{-q(d)}G(q)$$

1.4 GT of Derivatives of $g(t)$:

$$\dot{R}\{g'(t)\} = qG(q) - \frac{1}{q^3}g(0),$$

$$\dot{R}\{g''(t)\} = q^2G(q) - \frac{1}{q^2}g(0) - \frac{1}{q^3}g'(0)$$

and so on.

2. Methodology

2.1. Undamped Mechanical Oscillator

The differential equation of the undamped mechanical oscillator [5], [6] subjected to a triangular pulse force is given by

$$m\ddot{y}(t) + ky(t) = F_o\left(1 - \frac{t}{t_1}\right)$$

Or

$$\ddot{y}(t) + \omega_0{}^2y(t) = \frac{F_o}{m}\left(1 - \frac{t}{t_1}\right) \quad \text{…. (1)}$$

$where\ \omega_0 = \sqrt{\frac{k}{m}}$, $F_o(1 - \frac{t}{t_1})$ is a triangular pulse force, [7] y (0) = 0 and $\dot{y}(0) = 0$.

The GT of (1) provides

$$q^2\bar{y}(q) - \frac{1}{q^2}y(0) - \frac{1}{q^3}\dot{y}(0) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}\frac{1}{q^3}\int_0^\infty e^{-qt}\left(1 - \frac{t}{t_1}\right)dt$$

$$q^2\bar{y}(q) - \frac{1}{q^2}y(0) - \frac{1}{q^3}\dot{y}(0) + \omega_0{}^2\bar{y}(q) = \}\frac{F_o}{m}\frac{1}{q^3}\int_0^{t_1} e^{-qt}\left(1 - \frac{t}{t_1}\right)dt + q^3\int_{t_1}^\infty e^{-qt}\,(0)dt$$

Here $\bar{y}(q)$ denotes the GT of y(t).

Put $y(0) = 0$ and $\dot{y}(0) = 0$ and simplifying (2), we get

$$= \frac{F_o}{m}\{\frac{1}{q^3}\int_0^{t_1} e^{-qt}\,(1)\,dt q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) - \frac{1}{q^3}\int_0^{t_1} e^{-qt}\left(\frac{t}{t_1}\right)dt\}$$

$$q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}\{-\frac{1}{q^4}[e^{-qt_1} - 1] + \frac{1}{q^4}[e^{-qt_1}] - \frac{1}{q^4t_1}\int_0^{t_1} e^{-qt}\,dt\}$$

$$q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}\{-\frac{1}{q^4}[e^{-qt_1} - 1] + \frac{1}{q^4}[e^{-qt_1}] + \frac{1}{q^5t_1}[e^{-qt_1} - 1]\}$$

$$q^2\bar{y}(q) + \omega_0{}^2\bar{y}(q) = \frac{F_o}{m}\left[\frac{1}{q^4} + \frac{1}{q^5t_1}e^{-qt_1} - \frac{1}{q^5t_1}\right]$$

$$\bar{y}(q) = \frac{F_o}{m}\frac{1}{q^3}\left(\frac{1}{q(q^2+\omega_0{}^2)} - \frac{1}{t_1q^2(q^2+\omega_0{}^2)} + \frac{e^{-qt_1}}{t_1q^2(q^2+\omega_0{}^2)}\right)$$

$$\bar{y}(q) = \frac{F_o}{m}\frac{1}{q^3}\{\frac{1}{q(\omega_0{}^2)} - \frac{q}{(\omega_0{}^2)(q^2+\omega_0{}^2)} - \frac{1}{t_1\,q^2(\omega_0{}^2)} + \frac{1}{t_1(\omega_0{}^2)(q^2+\omega_0{}^2)} + \frac{e^{-qt_1}}{t_1\,q^2(\omega_0{}^2)} - \frac{e^{-qt_1}}{t_1(\omega_0{}^2)(q^2+\omega_0{}^2)}\}$$

$$\bar{y}(q) = \frac{F_o}{m}\{\frac{1}{q^4(\omega_0{}^2)} - \frac{1}{(\omega_0{}^2)\,q^2(q^2+\omega^2)} - \frac{1}{t_1\,q^5(\omega_0{}^2)} + \frac{1}{t_1(\omega_0{}^2)\,q^3(q^2+\omega^2)} + \frac{e^{-qt_1}}{t_1\,q^2(\omega_0{}^2)} - \frac{e^{-qt_1}}{t_1(\omega_0{}^2)(q^2+\omega_0{}^2)}\}$$

Taking inverse GT, we have

$$y(t) = \frac{F_o}{m}\{\frac{1}{(\omega_0{}^2)} - \frac{\cos\omega_0 t}{(\omega_0{}^2)} - \frac{t}{t_1(\omega_0{}^2)} + \frac{\sin\omega_0 t}{t_1\omega_0(\omega_0{}^2)} + \frac{t-t_1}{t_1(\omega_0{}^2)}U(t-t_1) - \frac{\sin\omega_0(t-t_1)}{t_1\omega_0(\omega_0{}^2)}U(t-t_1)\}$$

$$y(t) = \frac{F_o}{m(\omega_0{}^2)}\{1 - \cos\omega_0 t - \frac{t}{t_1} + \frac{\sin\omega_0 t}{t_1\omega_0} + \frac{t-t_1}{t_1(\omega_0{}^2)}U(t-t_1) - \frac{\sin\omega_0(t-t_1)}{t_1\omega_0(\omega_0{}^2)}U(t-t_1)\}$$

$$y(t) = \frac{F_o}{k}\{\frac{\sin\omega_0 t}{t_1\omega_0} - \cos\omega_0 t + 1 - \frac{t}{t_1} + \frac{t-t_1}{t_1(\omega_0{}^2)}U(t-t_1) - \frac{\sin\omega_0(t-t_1)}{t_1\omega_0(\omega_0{}^2)}U(t-t_1)\}\ldots (3)$$

For t < t_1,

$$y(t) = \frac{F_o}{k}\{\frac{\sin\omega_0 t}{t_1\omega_0} - \cos\omega_0 t + \left(1 - \frac{t}{t_1}\right) \ldots (4a)$$

For t > t_1,

$$y(t) = \frac{F_o}{k}\{\frac{\sin\omega_0 t}{t_1\omega_0} - \cos\omega_0 t - \frac{\sin\omega(t-t_1)}{t_1\omega_0}\}\ldots (4b)$$

The graph of equation (3) (taking $t_1 = 1s, \frac{F_o}{k} = 1000m, \omega_0 = 314\ rad/s$) is shown in figure 2.

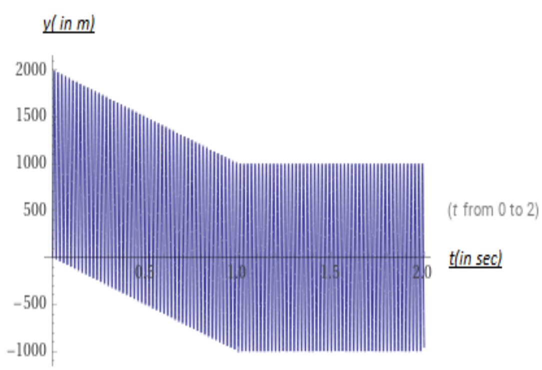

Figure 2: Response of an undamped mechanical oscillator exposed to a triangular pulse force

2.2. Undamped Electrical Oscillator

The differential equation of the electrical oscillator subjected to a triangular pulse force potential [8, 9] is given by

$$L\ddot{Q}(t) + \frac{1}{C}Q(t) = V_o\left(1 - \frac{t}{t_1}\right)$$

Or

$$\ddot{Q}(t) + \omega_0{}^2 Q(t) = \frac{V_o}{L}(1 - \frac{t}{t_1}) \ \ldots (5)$$

where $\omega_0 = \sqrt{\frac{1}{LC}}$, $V_o(1 - \frac{t}{t_1})$ is a triangular pulse potential, Q (0) = 0 and $\dot{Q}(0) = 0$.

The GT of equation (5) provides

$$q^2\bar{Q}(q) - \frac{1}{q^2}Q(0) - \frac{1}{q^3}\dot{Q}(0) + \omega_0{}^2\bar{Q}(q) = \frac{V_o}{L}\frac{1}{q^3}\int_0^\infty e^{-qt}\left(1 - \frac{t}{t_1}\right)dt$$

$$q^2\bar{Q}(q) - \frac{1}{q^2}Q(0) - \frac{1}{q^3}\dot{Q}(0) + \omega_0{}^2\bar{Q}(q) = \frac{V_o}{L}\{\frac{1}{q^3}\int_0^{t_1} e^{-qt}\left(1 - \frac{t}{t_1}\right)dt + \frac{1}{q^3}\int_{t_1}^\infty e^{-qt}(0)dt\}$$

Here $\bar{Q}(q)$ denotes the GT of Q(t).

Put $Q(0) = 0$ and $\dot{Q}(0) = 0$ [10-12] and simplifying (5), we get

$$= \frac{V_o}{L}\{\frac{1}{q^3}\int_0^{t_1} e^{-qt}(1)\,dt - q^2\bar{Q}(q) + \omega_0{}^2\bar{Q}(q) \ \frac{1}{q^3}\int_0^{t_1} e^{-qt}\left(\frac{t}{t_1}\right)dt\}$$

$$q^2\bar{Q}(q) + \omega_0{}^2\bar{Q}(q) = \frac{V_o}{L}\{-\frac{1}{q^4}[e^{-qt_1} - 1] + \frac{1}{q^4}[e^{-qt_1}] - \frac{1}{q^4 t_1}\int_0^{t_1} e^{-qt}\,dt\}$$

$$q^2\bar{Q}(q) + \omega_0{}^2\bar{Q}(q) = \frac{V_o}{L}\{-\frac{1}{q^4}[e^{-qt_1} - 1] + \frac{1}{q^4}[e^{-qt_1}] + \frac{1}{q^5 t_1}[e^{-qt_1} - 1]\}$$

$$q^2\bar{Q}(q) + \omega_0{}^2\bar{Q}(q) = \frac{V_o}{L}\left[\frac{1}{q^4} + \frac{1}{q^5 t_1}e^{-qt_1} - \frac{1}{q^5 t_1}\right]$$

$$\bar{Q}(q) = \frac{V_o}{L}\frac{1}{q^3}\left(\frac{1}{q(q^2 + \omega_0{}^2)} - \frac{1}{t_1 q^2(q^2 + \omega_0{}^2)} + \frac{e^{-qt_1}}{t_1 q^2(q^2 + \omega_0{}^2)}\right)$$

$$\bar{Q}(q) = \frac{V_o}{L}\frac{1}{q^3}\{\frac{1}{q(\omega_0{}^2)} - \frac{q}{(\omega_0{}^2)(q^2 + \omega_0{}^2)} - \frac{1}{t_1 q^2(\omega_0{}^2)} + \frac{1}{t_1(\omega_0{}^2)(q^2 + \omega_0{}^2)} + \frac{e^{-qt_1}}{t_1 q^2(\omega_0{}^2)} - \frac{e^{-qt_1}}{t_1(\omega_0{}^2)(q^2 + \omega_0{}^2)}\}$$

$$\bar{Q}(q) = \frac{V_o}{L}\{\frac{1}{q^4(\omega_0{}^2)} - \frac{1}{(\omega_0{}^2)\,q^2(q^2 + \omega^2)}$$

$$-\frac{1}{t_1\, q^5(\omega_0{}^2)}+\frac{1}{t_1(\omega_0{}^2)\, q^3(\, q^2+\ \omega^2)}$$

$$+\frac{e^{-qt_1}}{t_1\, q^2(\omega_0{}^2)}-\frac{e^{-qt_1}}{t_1(\omega_0{}^2)(\, q^2+\ \omega_0{}^2)}\}$$

Taking inverse GT, we have

$$Q(\mathrm{t})=\ \frac{V_o}{L}\{\frac{1}{(\omega_0{}^2)}-\frac{\cos\omega_0 t}{(\omega_0{}^2)}-\frac{t}{t_1(\omega_0{}^2)}+\frac{\sin\omega_0 t}{t_1\omega_0(\omega_0{}^2)}+\frac{t-t_1}{t_1(\omega_0{}^2)}U(t-t_1)-\frac{\sin\omega_0(t-t_1)}{t_1\omega_0(\omega_0{}^2)}U(t-t_1)\}$$

$$Q(\mathrm{t})=\ \frac{V_o}{L(\omega_0{}^2)}\{1-\cos\omega_0 t-\frac{t}{t_1}+\frac{\sin\omega_0 t}{t_1\omega_0}+\frac{t-t_1}{t_1(\omega_0{}^2)}U(t-t_1-\frac{\sin\omega_0(t-t_1)}{t_1\omega_0(\omega_0{}^2)}U(t-t_1)\}$$

$$Q(\mathrm{t})=\ V_o C\{\frac{\sin\omega_0 t}{t_1\omega_0}-\cos\omega_0 t+1-\frac{t}{t_1}+\frac{t-t_1}{t_1(\omega_0{}^2)}U(t-t_1)-\frac{\sin\omega_0(t-t_1)}{t_1\omega_0(\omega_0{}^2)}U(t-t_1)\}\ldots (6)$$

For t < t_1,

$$Q(\mathrm{t})=\ V_o C\{\frac{\sin\omega_0 t}{t_1\omega_0}-\cos\omega_0 t+\left(1-\frac{t}{t_1}\right)\ldots (7a)$$

For t > t_1,

$$Q(\mathrm{t})=\ V_o C\{\frac{\sin\omega_0 t}{t_1\omega_0}-\cos\omega_0 t-\frac{\sin\omega(t-t_1)}{t_1\omega_0}\}\ldots (7b)$$

The graph of equation (6) (taking $t_1=1s, \frac{V_o}{k}=500m, \omega_0=314\ rad/s$) is shown in figure 3 below.

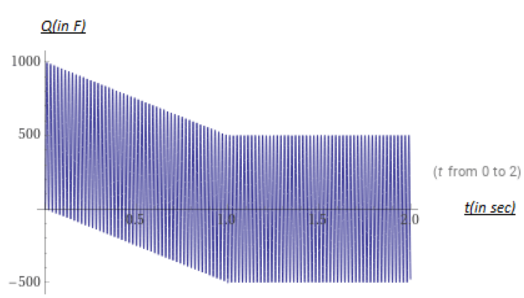

Figure 3: Response of an undamped electrical oscillator exposed to a triangular pulse potential.

3. Results and Discussion

It is clear from Figure 2 that due to triangular pulse, the response (displacement) of an undamped mechanical oscillator exposed to a triangular pulse first increases towards right side (say) with large amplitude and then decreases and becomes zero and then increases towards left side with small amplitude and then decreases and becomes zero. Again, it repeats the same behavior again and again till the effect of triangular pulse becomes zero, but with linearly decreasing amplitude towards right side and correspondingly linearly increasing amplitude towards the left side. As soon as the effect of triangular pulse becomes zero, the nature of displacement of an undamped mechanical oscillator exposed to a triangular Pulse suddenly becomes oscillatory with constant amplitude. It is clear from Figure 3 that due to triangular pulse, the response (electric charge) of an undamped electrical oscillator exposed to a triangular pulse first increases in one direction with large amplitude and then decreases and becomes zero and then increases towards opposite side with small amplitude and then decreases and becomes zero. Again, it repeats the same behaviour again and again till the effect of the triangular pulse becomes zero, but with linearly decreasing amplitude in one direction and correspondingly linearly increasing amplitude in the other direction. As the triangular pulse ceases, the response of an undamped electrical oscillator exposed to a triangular Pulse suddenly becomes oscillatory with constant amplitude.

4. Conclusion

In this paper, the response of an undamped (mechanical as well as electrical) oscillator subjected to a triangular pulse was successfully determined by the Gupta integral transform (GT). This paper exemplified the GT for determining the response of an undamped mechanical oscillator as well as an electrical oscillator subjected to a triangular pulse and proves that GT is more effective method than calculus.

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