Solving Advanced Missile Robotic Control Problem Via Integral Rohit transform

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ABSTRAC: The problem of robotic control of an advanced missile requires designing control systems for remote-controlled missiles with sufficient abilities. These control systems need to handle navigation, guidance, propulsion, and potentially other tasks like target acquisition and avoidance of obstacles. The various aspects of missile dynamics are described by complex and non-linear differential equations. Due to this, it becomes difficult to obtain their analytical solutions. During system design and analysis, to solve these complex and non-linear differential equations, numerical techniques like Euler's technique, the Runge-Kutta technique, or more advanced mathematical techniques are generally used. The advanced missile robotic control problem is one of the problems represented by second-order linear differential equations. In this paper, the integral Rohit transform (RT) has been applied to solve an advanced missile robotic control problem. This transformation has never been applied in working out the advanced missile robotic control problem. This paper applies a new technique for solving the second-order linear differential equation representing the advanced missile robotic control problem to obtain its required turn angle so that it will be able to catch and destroy the attacker. The necessary graphs are plotted to illustrate the numerical solution of the robotic control of an advanced missile. It reveals that the integral Rohit transform (RT) technique is an effective technique to solve the advanced missile robotic control problem.

Keywords: Advanced missile robotic control problem; Integral Rohit transform, Second-order linear differential equation.

1. Introduction

The problem of robotic control of an advanced missile requires designing control systems for remote-controlled missiles with sufficient abilities [1]. These control systems are required to handle navigation, guidance, propulsion, and other jobs like target acquisition, avoidance of obstacles, real-time decision making, communication, and machine learning algorithms, and vigorous control techniques are used to address these jobs. Before deployment, simulation and testing are also necessary to confirm the performance of the control system. The motion of an advanced missile robotic control problem is governed by the differential equations which rely on the particular dynamics of the system. Generally, the concepts of mechanics and aerodynamics [3] are called for the equations of motion for an advanced missile. The equations of kinematics explain the motion of the advanced missile in terms of its position, velocity, and acceleration. The forces acting on the missile are related to its acceleration by the Newton's second law of motion. The aero-dynamic forces are set out by the aero-dynamic equations. These forces depend on its shape, air speed, angle of attack, and other factors. The differential equations governing the propulsion system of the missile show how thrust is generated as a function of engine parameters. Differential equations governing the guidance and control systems measure how the missile adjusts its trajectory to reach the desired target [4]. The various aspects of missile dynamics are described by complex and non-linear differential equations. As a result, the analytical solutions become difficult or impossible to obtain. During system design and analysis, the numerical techniques or more advanced mathematical techniques are generally used to solve these complex and non-linear differential equations. One of the applications of second-order linear differential equations is the advanced missile robotic control problem. Differential equations represent the mathematical models in Physics, Medical, Engineering, Economics, and Chemistry. There are many practices to solve these differential equations. Laplace transform has been brought to bear extensively in solving problems expressed by differential equations [1-4]. However, integral Rohit transform has not been sufficiently brought to bear in such problems due to its recent appearance. The author Rohit Gupta has proffered the integral Rohit transform in recently to expedite the technique of working out differential equations. It has been previously applied to solve problems represented by differential equations in science and engineering [5-10]. The paper applies the integral Rohit transform for working out advanced missile robotic control problem represented by second order linear differential equation.

2. Basics of Rohit Transform

The integral Rohit transform (RT) of a function g(t) is defined [6] by the following integral equations.

$R\{h(t)\} = q^3 \int_0^\infty e^{-qt} h(t)dt,\ t \geq 0,\ q_1 \leq q \leq q_2$. The main purpose for using integral Rohit transform for finding the solution of advanced missile robotic control problem is that the application of Rohit transform converts such problems to simple algebraic problem.

The Rohit transforms (RTs) of unidentified functions [7] are given by

- ❖ $R\{t^n\} = \frac{n!}{q^{n-2}}$, $where\ n\ is\ 0, 1, 2 \ldots.$
- ❖ $R\{sinbt\} = \frac{b\,q^3}{q^2+b^2}$,
- ❖ $R\{cosbt\} = \frac{q^4}{q^2+b^2}$,

The Rohit transforms (RTs) of unidentified derivatives [8, 9] are

$R\{h'(t)\} = qR\{h(t)\} - q^3h(0)$,

$R\{h''(t)\} = q^2R\{h(t)\} - q^4h(0) - q^3h'(0)$, and so on.

Highlights

- ✓ The RT technique is proffered for handing out the advanced missile robotic control problem.
- ✓ The differential equation representing the advanced missile robotic control problem is solved without first finding a general solution.
- ✓ Highly subtle and rigorous results are obtained.
- ✓ The paper disports with no doubt that the integral Rohit transform is an effective mathematical technique to solve the advanced missile robotic control problem.

3. Methodology

Let us first consider the typical missile robotic control problem. Consider a missile B that is seeking out an attacker aircraft. If the attacker aircraft turns by an unidentified angle $\emptyset(t)$ at any time, then the missile B must also turn by the matching angle, if it has to grab and knock down the attacker. Some guiding system is needed to cause the desired turns, but since the missile has to be done this immediately, therefore, there is a need of for a substitute in the hands of the person which will turn the shaft by some angle in order to cause the necessary turns. In this application, let us suppose that the necessary angle of turn as specified by the radar beam be αt and $\emptyset(t)$ be the angle of turn of the shaft at time t. Because the activities are occurring quickly, therefore, there may be an error between the angle of turn of the shaft and the desired angle of turn i.e. error = $\emptyset(t) - \alpha t$. The existence of the error must be signaled back to the shaft, so that a compensating turning effects or torque be produced. If the error is large, the torque needed will be large. If the error is small, the torque needed will be small. Hence the required torque is proportional to the error.

The second-order linear differential equation with constant coefficients representing the typical missile robotic control problem is written as [11, 12]:

$$I\ddot{\emptyset}(t) + \delta\emptyset(t) = \delta\alpha t$$

Or

$$\ddot{\emptyset}(t) + \frac{\delta}{I}\emptyset(t) = \frac{\delta}{I}\alpha t \qquad (1)$$

Here $\emptyset(t)$ is the turning angle of the aircraft at any instant t, $\ddot{\emptyset}(t)$ is the change in angular velocity per unit time, αt is the assumed desired turning angle of the ballistic missile, $\boldsymbol{\alpha}$ is the change in angular displacement per unit time of the ballistic missile, $\boldsymbol{I}$ is the MOI (moment of inertia) and $\delta(> 0)$ is a constant. Further, the initial turning angle is assumed to be $\emptyset(0) = 0$ and initial change in angular displacement per unit time $\dot{\emptyset}(0) = 0$.

The Rohit transform of Equation (1) provides

$$q^2\overline{\emptyset}(q) - q^4\emptyset(0) - q^3\dot{\emptyset}(0) + \frac{\delta}{I}\overline{y}(q) = \frac{\delta}{I}\alpha q \qquad (2)$$

$\overline{\emptyset}(q)$ implies the RT of $\emptyset(t)$.

Substituting $\emptyset(0) = 0$ and $\dot{\emptyset}(0) = 0$ and arranging Equation (2), we get

$$q^2\overline{\emptyset}(q) + \frac{\delta}{I}\overline{\emptyset}(q) = \frac{\delta}{I}\alpha q$$

$$\overline{\emptyset}(q) = \frac{\delta\alpha}{I}\{\frac{q}{\left(q^2+\frac{\delta}{I}\right)}\}$$

$$\overline{\emptyset}(q) = \frac{\delta\alpha}{I}\{\ \frac{q}{\frac{\delta}{I}} - \frac{q^3}{\frac{\delta}{I}\left(q^2+\frac{\delta}{I}\right)}\}$$

$$\overline{\emptyset}(q) = \alpha\{\ q - \frac{q^3}{\left(q^2+\frac{\delta}{I}\right)}\}$$

The above equation can be rewritten as:

$$\overline{\emptyset}(q) = \alpha\{q - \frac{\sqrt{\frac{\delta}{I}}q^3}{\sqrt{\frac{\delta}{I}}\left(q^2+\sqrt{\frac{\delta}{I}}^2\right)}\}$$

Taking inverse RT, we have

$$\emptyset(t) = \alpha\{t - \frac{\sin\sqrt{\frac{\delta}{I}}t}{\sqrt{\frac{\delta}{I}}}\}$$

This is the required turn at any instant t.

Assuming that $\sqrt{\frac{\delta}{I}} = 0.01\ and\ \alpha = 10$, the numerical solution of Equation (1) is conveyed in Figure 1.

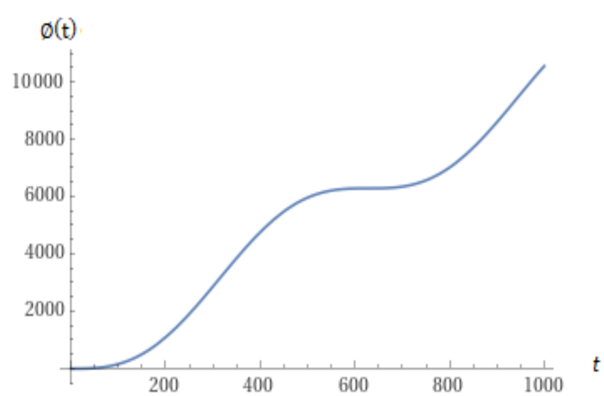

Figure 1: Numerical solution of Equation (1)

Let us now consider the advanced missile robotic control problem. The second-order linear differential equation with variable coefficients representing the advanced missile robotic control problem is written as [13]:

$t^2 I\ddot{\emptyset}(\mathrm{t}) + \delta\emptyset(\mathrm{t}) = \delta\alpha t$

Or

$$t^2\ddot{\emptyset}(\mathrm{t}) + \frac{\delta}{\mathrm{I}}\emptyset(t) = \frac{\delta}{\mathrm{I}}\alpha t \qquad (3)$$

The Rohit transform (RT) [14] of Equation (3) provides

$q^2\emptyset''(q) - 2r\emptyset(\mathrm{q}) + \left(2 + \frac{\delta}{\mathrm{I}}\right)\emptyset(\mathrm{q}) = \frac{\delta}{\mathrm{I}}\alpha q$

On putting, q = e^x, the above Equation is modified to

$$\emptyset''(\mathrm{x}) - 3\emptyset'(\mathrm{x}) + \left(2 + \frac{\delta}{\mathrm{I}}\right)\emptyset(\mathrm{x}) = \frac{\delta}{\mathrm{I}}\alpha e^x \qquad (4)$$

The solution of homogeneous equation:

$\emptyset''(\mathrm{x}) - 3\emptyset'(\mathrm{x}) + \left(2 + \frac{\delta}{\mathrm{I}}\right)\emptyset(\mathrm{x}) = 0$, is given by

$$\emptyset(\mathrm{x}) = Ae^{-\frac{1}{2}\left[\sqrt{1-4\frac{\delta}{\mathrm{I}}}-3\right]x} + Be^{\frac{1}{2}\left[\sqrt{1-4\frac{\delta}{\mathrm{I}}}+3\right]x}$$

$$\emptyset(\mathrm{x}) = Ae^{-cx} + Be^{fx}$$

Where, $c = \frac{1}{2}\left[\sqrt{1-4\frac{\delta}{\mathrm{I}}} - 3\right]$ and $f = \frac{1}{2}\left[\sqrt{1-4\frac{\delta}{\mathrm{I}}} + 3\right]$

The partial integral is given by

P.I.= $e^{-cx}u + e^{fx}v$

$$Where\ u = -\int \frac{e^{fx}\frac{\delta}{\mathrm{I}}\alpha e^x}{\left[e^{-cx}\frac{d}{dx}e^{fx} - e^{fx}\frac{d}{dx}e^{-cx}\right]}dx = -\frac{\delta}{\mathrm{I}}\frac{\alpha e^{(1+c)x}}{(c+f)(1+c)}$$

$$And\ v = \int \frac{e^{-cx}\frac{\delta}{\mathrm{I}}\alpha e^x}{\left[e^{-cx}\frac{d}{dx}e^{fx} - e^{fx}\frac{d}{dx}e^{-cx}\right]}dx = -\frac{\delta}{\mathrm{I}}\frac{\alpha e^{(1-f)x}}{(c+f)(1-f)}$$

P.I.= $e^{-cx}u + e^{fx}v = -\frac{\delta}{\mathrm{I}}\frac{\alpha e^x}{(c+f)(1+c)} - \frac{\delta}{\mathrm{I}}\frac{\alpha e^x}{(c+f)(1-f)}$

$$P.I. = -\frac{\delta\alpha}{\mathrm{I}(c+f)}\left[\frac{1}{(1+c)} + \frac{1}{(1-f)}\right]e^x$$

$$P.I. = \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}e^x$$

Thus, the solution of equation (4) is given by

$$\emptyset(\mathrm{x}) = Ae^{-cx} + Be^{fx} + \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}e^x$$

Replacing x by logq, we have

$$\emptyset(\mathrm{q}) = Ae^{-c\log\mathrm{q}} + Be^{f\log\mathrm{q}} + \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}e^{\log\mathrm{q}}$$

$$\emptyset(\mathrm{q}) = Aq^{-c} + Bq^f + \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}q \qquad (5)$$

Taking inverse RT, we have

$$\emptyset(\mathrm{t}) = \frac{At^{c+2}}{\Gamma(c+3)} + \frac{Bt^{-f+2}}{\Gamma(-f+3)} + \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}t \qquad (6)$$

To find the constants A and B, we will apply conditions: $\emptyset(1) = 0\ and\ \emptyset'(1) = 0.$

On applying these conditions to equation (6) and solving, we obtain:

$$A = \frac{-\alpha(-1+f)\Gamma(c+3)}{1-4\frac{\delta}{\mathrm{I}}}\ and\ B = \frac{-\alpha(1+c)\Gamma(-f+3)}{1-4\frac{\delta}{\mathrm{I}}}$$

Hence equation (6) becomes

$$\emptyset(\mathrm{t}) = -\frac{\alpha(-1+f)}{1-4\frac{\delta}{\mathrm{I}}}t^{c+2} - \frac{\alpha(1+c)}{1-4\frac{\delta}{\mathrm{I}}}t^{-f+2} + \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}t$$

$$\emptyset(\mathrm{t}) = \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}t - \frac{\alpha(-1+f)}{1-4\frac{\delta}{\mathrm{I}}}t^{c+2} - \frac{\alpha(1+c)}{1-4\frac{\delta}{\mathrm{I}}}t^{-f+2}$$

$$\emptyset(\mathrm{t}) = \frac{\alpha}{\sqrt{1-4\frac{\delta}{\mathrm{I}}}}t + \frac{\alpha}{2}\frac{1}{1-4\frac{\delta}{\mathrm{I}}}\left[5 - \sqrt{1-4\frac{\delta}{\mathrm{I}}}\right]t^{\frac{1}{2}\left[7+\sqrt{1-4\frac{\delta}{\mathrm{I}}}\right]} - \frac{\alpha}{2}\frac{1}{1-4\frac{\delta}{\mathrm{I}}}\left[5 + \sqrt{1-4\frac{\delta}{\mathrm{I}}}\right]t^{\frac{1}{2}\left[7-\sqrt{1-4\frac{\delta}{\mathrm{I}}}\right]}$$

This is the required turn at any instant t.

Assuming that $\frac{\delta}{\mathrm{I}} = 0.01\ and\ \alpha = 10,$ the numerical solution of equation (3) is conveyed in Figure 2.

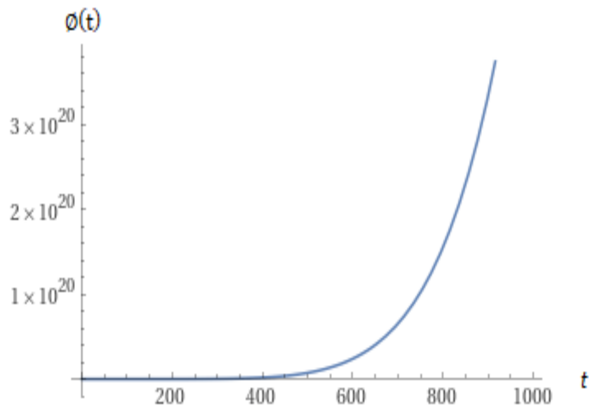

Figure 2: Numerical solution of Equation (3)

4. Conclusion

The advanced missile robotic control problem represented by the second-order linear differential equation has been solved fruitfully utilizing the integral Rohit transform and obtained the required turning angle of the missile at any instant t, so that it will be able to catch and destroy the attacker. It demonstrated that it is possible to utilize RT as a solving technique for the advanced missile robotic control problem. The numerical solution of the problem is provided by plotting the graphs which reveal that the proposed technique is an effective tool for solving advanced missile robotic control problem. It proved the ability of RT to solve advanced missile robotic control problems.

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References

[1] Said-Houari, B., 2015, Differential Equations: Methods and Applications, Springer International Publishing: Switzerland.

[2] John, H., 1991, Automatic Control of Aircraft and Missiles,. 2nd edition, Wiley: Canada.

[3] Gopal, Control Systems Principles and Design, 2008. 3rd edition, Tata McGraw Hill Publication.

[4] Palani, S., 2021, Automatic Control System,. 2nd Ed. Ane Books Pvt. Ltd.

[5] Gupta, R., 2020, On novel integral transform: Rohit Transform and its application to boundary value problems, ASIO Journal of Chemistry, Physics, Mathematics and Applied Sciences (ASIO-JCPMAS), 4(1), 08-13. http://doi-ds.org/doilink/06.2020-62339259/

[6] Gupta, R., Singh, I., and Sharma, A., 2022, Response of an Undamped Forced Oscillator via Rohit Transform," International Journal of Emerging Trends in Engineering Research, 10(8), 396-400. https://doi.org/10.30534/ijeter/2022/031082022

[7] Gupta, R., 2023, Mechanically Persistent Oscillator Supplied with Ramp Signal, Al-Salam Journal for Engineering and Technology, 2(2), 112–115. https://doi.org/10.55145/ajest.2023.02.02.014

[8] Gupta, R., Gupta, R., Kumar, A., 2022, Analysis of Impulsive Response of Mechanical and Electrical Oscillators by Rohit Transform, Engineering and Scientific International Journal (ESIJ), 9(3), 51-54.

[9] Gupta, R., Sharma, A., 2023, Response of Non-Damped Oscillators Subjected to Rectangular Pulse, International Journal of Engineering and Applied Physics, 3(3), 858–864.

[10] Gupta, R., Gupta, R., Verma, D., 2022, Response of RLC network circuit with steady source via Rohit transform, International Journal of Engineering, Science and Technology,14(1), 21-27. http://dx.doi.org/10.4314/ijest.v14i1.3\

[11] Nagrath, I. J., and Gopal, M., 2021, Control System Engineering, New age international Publication. 7th Ed.

[12] Sawant, L. S., 2018, Applications of Laplace Transform in Engineering Fields, International Research, Journal of Engineering and Technology, 5(5), 3100-3105.

[13] Kuffi, E., Abbas, E.S., Mebayet, S.O., 2018, AI-Tememe transform in solving developed missile automatic control problem, International Journal of Engineering &Technology, 7(4), 6565-6567.

[14] Gupta, R., & Mahajan, D., 2023, Solving Problems of Physical or Basic Sciences via Rohit Transform. Al-Salam Journal for Engineering and Technology, 3(1), 109–116. https://doi.org/10.55145/ajest.2024.03.01.009