Spectral dispersion and thermo optic coefficient for Silicon material

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ABSTRAC: In this study, the first derivatives of this equation with respect to temperature (thermo-optic coefficients) and wavelength (spectral dispersion) were derived in the MATLAB software, and a special code was written to solve the equations and found for temperatures that range from 100 to 750 K and wavelength range from 1.2 to 14 µm. The results of the computation were compared with previously published data, and showed a good agreement. Once that was done, the values of phase velocity and group velocity that depend on refractive index and spectral dispersion in the medium were found. The phase velocity and group velocity were plotted in three dimensions to show the overall change over different values of wavelengths and temperature.

***Keywords*:** Refractive index, Silicon, thermo optics coefficient, spectral dispersion, group velocity.

1. Introduction

The spectral dispersion and thermo optic coefficient are important characteristics of medium that describe how light travels across a material at various wavelengths and temperatures[1].These characteristics are important in several applications like optical switches , modulators[2] and Photonic integrated circuits[3] .The latter characteristics were aid in enhancing performance and creating new applications for Silicon in the infrared wavelength region in other optical devices.

Thermo-optic coefficients (dn/dT) have been extensively studied in the ranges of 100–750 K and spectral dispersion at 293K by using the first derivative of the temperature-related Sellmeier equation [4] determined the thermo-optic material properties of the laser host materials using interference, laser performance modelling at 1064 nm in the range of temperatures (80–300 K) [5]. Bradley J. F. et al. observed dispersion and dn/dT at temperatures between (20 and 300) K and wavelengths between 1.1 and 5.6 µm [6] and used an interferometric technique to measure the thermo-optic coefficient of silicon (1550 nm). The results showed great agreement with earlier measurements above 30 K [7]. Derived and calculated spectral dispersion for Ge at temperatures between (20and300)K and wavelengths between 1.8 and 5.5 µm. The results were compared to earlier published data, and there was a good agreement with the work of Q.Esam [8].

2. Methodology (Experimental Procedure)

In an earlier work [9], we have derived that the Cauchy formula of eq. (1).Accordingly, the empirical data of ref [4] , and the results exhibited a good concurrence.

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \frac{D}{\lambda^6} \quad (1)$$

where n is the refractive index, λ is the wavelength, and A, B, C, and D are the parameters. Next, as shown in equation (2), by defining four parameters as a function of temperatures.

$$A = a_1T^3 + a_2T^2 + a_3T + a_4$$

$$B = b_1T^3 + b_2T^2 + b_3T + b_4$$

$$C = c_1T^3 + c_2T^2 + c_3T + c_4$$

$$D = d_1T^3 + d_2T^2 + d_3T + d_4 \quad (2)$$

It was found that the sixteen new parameters were obtained as shown in Table 1

Following the section that involves the study and calculation of the spectrum dispersion, thermo optic coefficient in addition to the group velocity.

3. Results and Discussion

1. Spectral dispersion

The relationship between $\boldsymbol{n}$ and (λ)in a dispersive medium is known as spectral dispersion. The value of spectral dispersion, dn/dλ:

$$\frac{\boldsymbol{dn(\lambda,T)}}{\boldsymbol{d\lambda}} = -2\frac{\boldsymbol{b_1T^3 + b_2T^2 + b_3T + b_4}}{\boldsymbol{\lambda^3}} - 4\frac{\boldsymbol{c_1T^3 + C_2T^2 + C_3T + C_4}}{\boldsymbol{\lambda^5}} - 6\frac{\boldsymbol{d_1T^3 + d_2T^2 + d_3T + d_4}}{\boldsymbol{\lambda^7}} \quad \textbf{(3)}$$

The spectral dispersion values rapidly reach 2.5µm ,became nearly constant. The spectral dispersion values for different temperatures were noticed to be close to each other, as shown in Figure**.**

Given that dn/dλ is non-zero, as in this paper for Silicon, in the case of normal dispersion, dn/dλ< 0, whereas in the event of anomalous dispersion, dn/dλ> 0. As the wavelength decreases, the dispersion increases, corresponding with the curve's negative slope.

2. Phase velocity

The phase velocity is defined as the velocity of a single-wavelength wave[10]. The formula for phase velocity vp is given in terms of angular frequency w and wave number k:

$$Vp = \frac{w}{k} = \frac{c}{n(\lambda,T)} = = \frac{c}{A(T) + \frac{B(T)}{\lambda^2} + \frac{C(T)}{\lambda^4} + \frac{D(T)}{\lambda^6}} \quad (4)$$

c : the speed of light, $3*10^8$ m/sec.

The phase velocity which was determined by Eq. (4), is illustrated in Figure. 5, this formula shows that the phase velocity is inversely proportional to temperature and directly proportional to wavelength.

3 .Group velocity

It was defined as the wave packet speed, where each wave has a different wavelength[10]. The relationship between the group velocity(vg) and the refractive- index and spectral dispersion [8] .

$$vg= \partial\omega/\partial k=c/((n-\lambda\ \partial n/\partial\lambda)) \qquad (5)[8]$$

$$Vg = (c/n)/(1-\lambda/n\ \ dn/d\lambda)=vp/(1-\lambda/n\ \ dn/d\lambda\ )$$

when values spectral dispersion are negative $dn/d\lambda>0$

$Vp>Vg$

The group velocity was calculated using E. 5. For temperatures range 100–750K, at a temperature of 293 K, it was compared to the values that were calculated by Equation 5 based on the values of the refractive index and spectral dispersion that were calculated by Li. The results showed a good agreement. From the results and Figures (6,7and 8) ,the group velocity increases rapidly for wavelengths between 1.2 μm and 2 μm, then increases very slowly until it approaches a constant value, which depends on the temperature and decreases with increasing temperature.

4. Temperature derivative of refractive index

It was showed that the temperature coefficient of refractive index can be calculated by deriving the temperature-dependent Cauchy equation Eq(2) in relation to temperature in the following way:

$$\frac{dn(\lambda,T)}{dT} = 3a_1T^2 + 2a_2T + a_3 + \frac{3b_1T^2 + 2b_2T + b_3}{\lambda^2} + \frac{3c_1T^2 + 2c_2T + c_3}{\lambda^4} + \frac{3d_1T^2 + 2d_2T + d_3}{\lambda^6} \qquad (6)$$

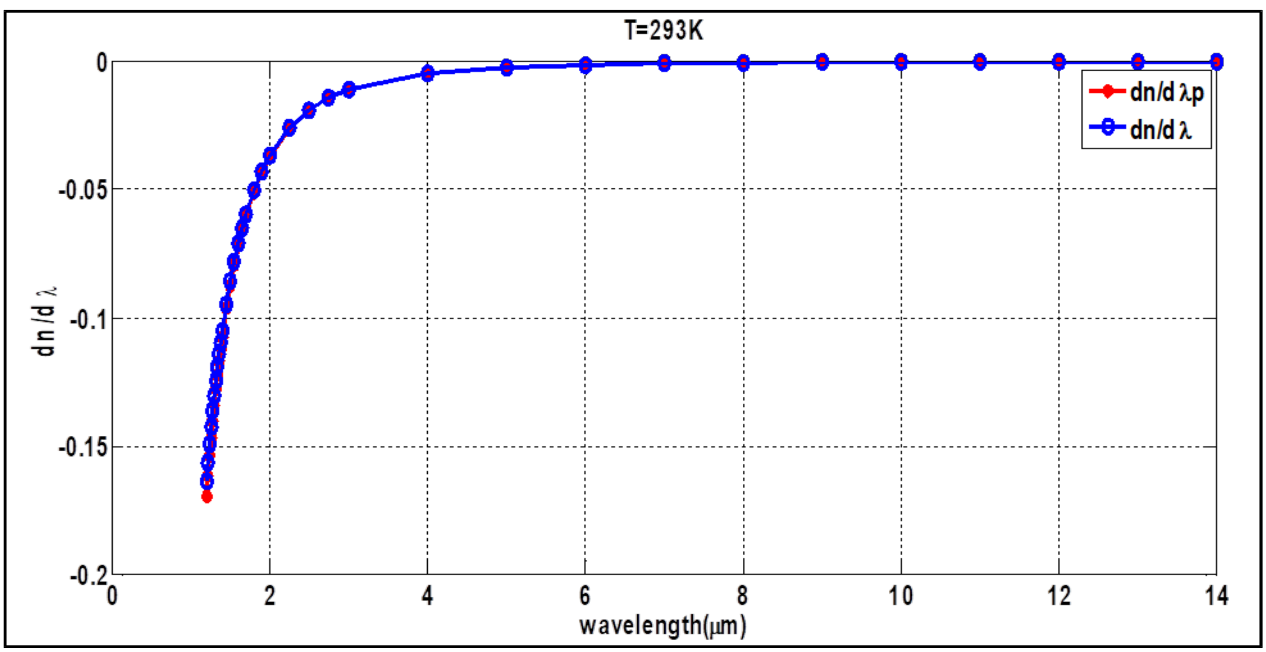
Using this formula, the temperature coefficient of Silicon was determined. The results were compared to the temperature coefficient values that the calculated for Li for temperatures that vary from 100 to 750 K [4]. It was found that the results showed a good agreement, reaching 3 digits at temperatures (400-750) K, with temperature accuracy (100-350K) 2 digits. This shows a good agreement for different values of temperatures as in show figures (9,10).

We observed from Figures 9 and 10 that the thermo optic coefficients calculated using

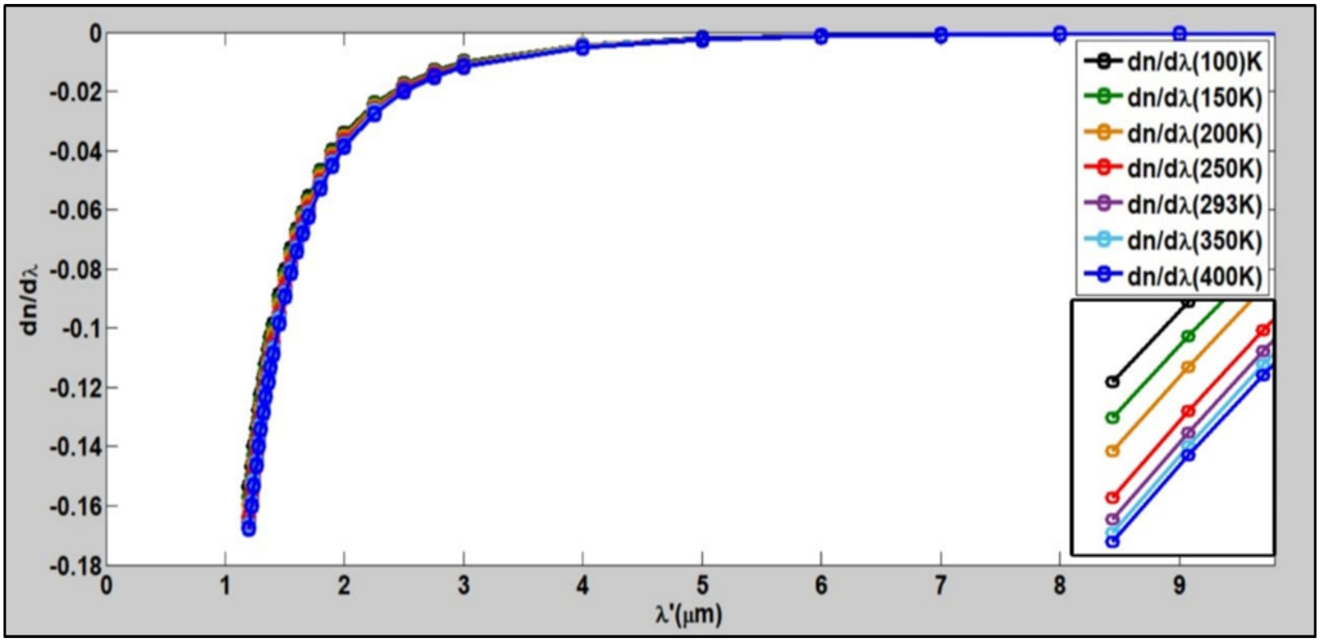
Equation 6 and the values from ref [4], is inversely proportional to the wavelength and directly proportional to the temperature.

Table 1: values of the parameters in Equation (2) [9].

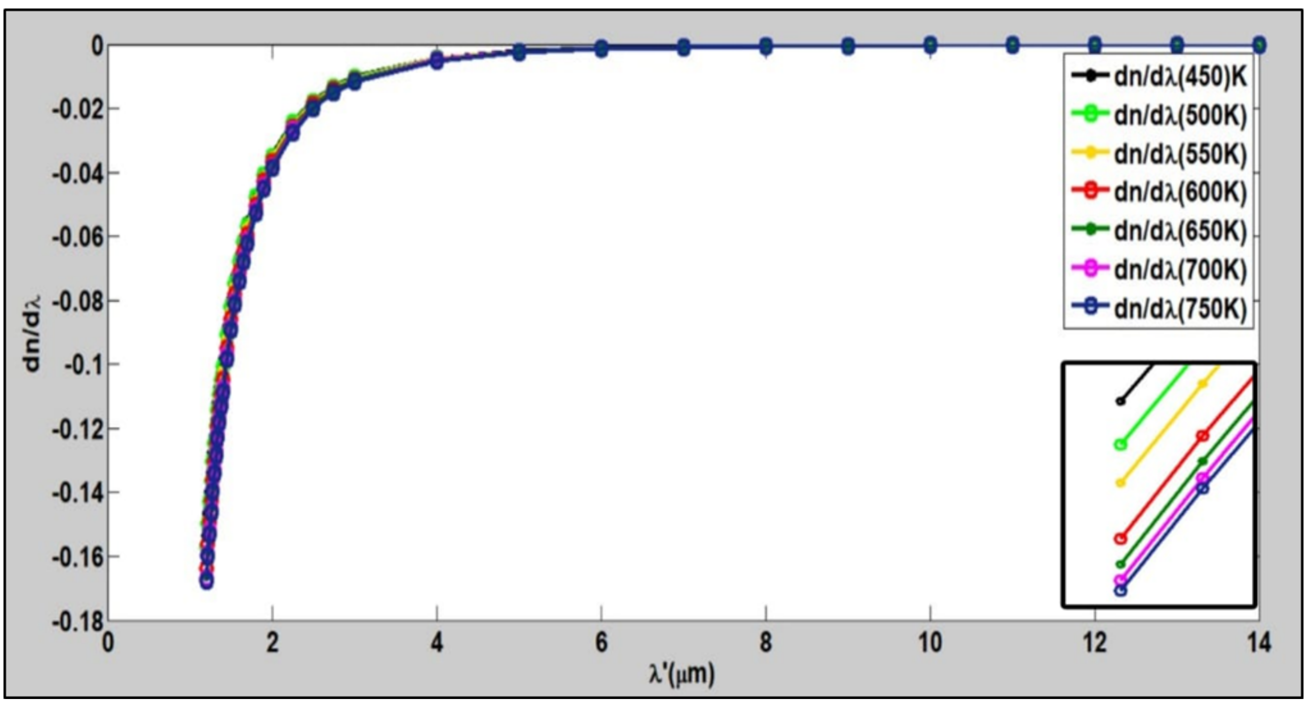
A		B		C		D	
a_1	$-1.15738*10^{-11}$	b_1	$-1.07728*10^{-11}$	c_1	$9.004653*10^{-11}$	d_1	$1.465861*10^{-10}$
a_2	$2.40175*10^{-7}$	b_2	$7.7650126*10^{-7}$	c_2	$1.459764*10^{-7}$	d_2	$-2.210402*10^{-7}$
a_3	$4.52707*10^{-5}$	b_3	$6.2628154*10^{-5}$	c_3	$7.38523*10^{-5}$	d_3	0.00010157442
a_4	3.38257	b_4	0.131866	c_4	0.00822973781	d_4	-0.0143774566



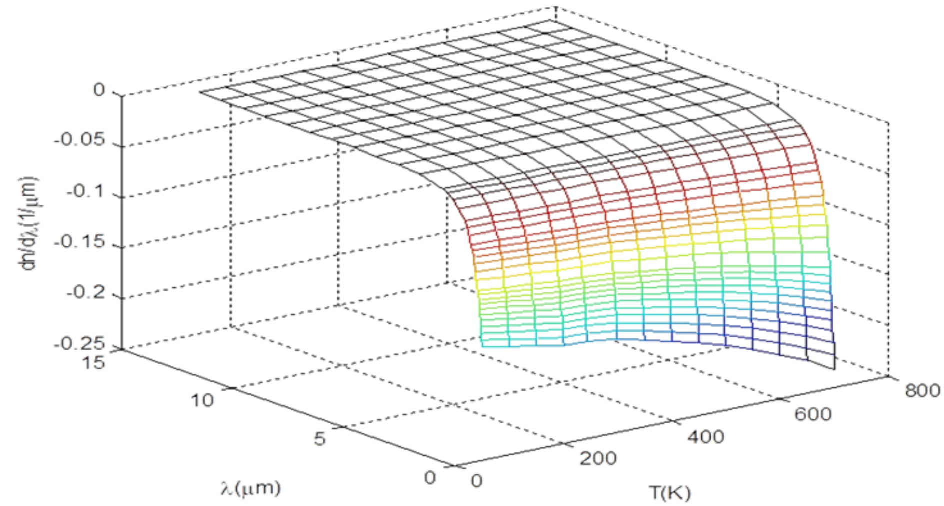
Fig(1): Spectral dispersion values were calculated using the empirical values dn/dλp in μm^{-1}. For temperature (293)K from ref.[4] and evaluated using Eq. (3), dn/dλ in μm^{-1}.



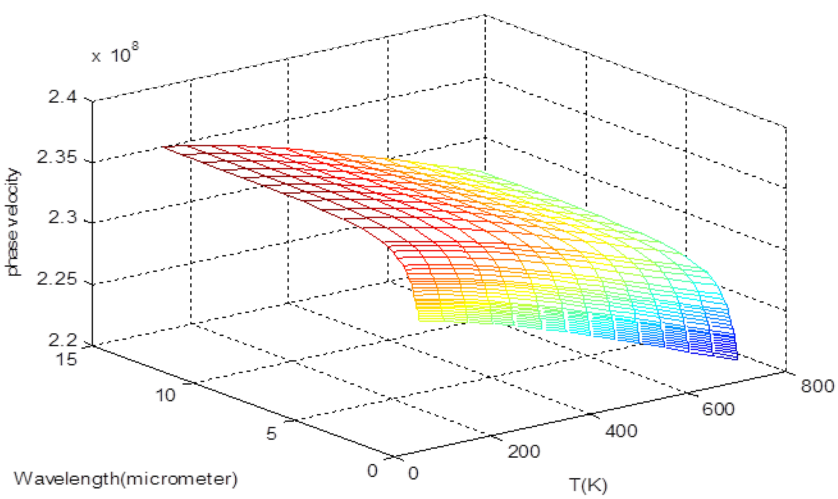
Fig(2):Spectral dispersion values were calculated using Eq. (3) and dn/dλ. At 100- 400 K in temperature.



Fig(3): Spectral dispersion values for temperature (450- 750 K) were calculated using Eq. (3).



Fig(4) : A 3-D figure displays how spectral dispersion dn/dλ depends on temperature and wavelength.



Fig(5): 3-D graphic represents phase velocity.

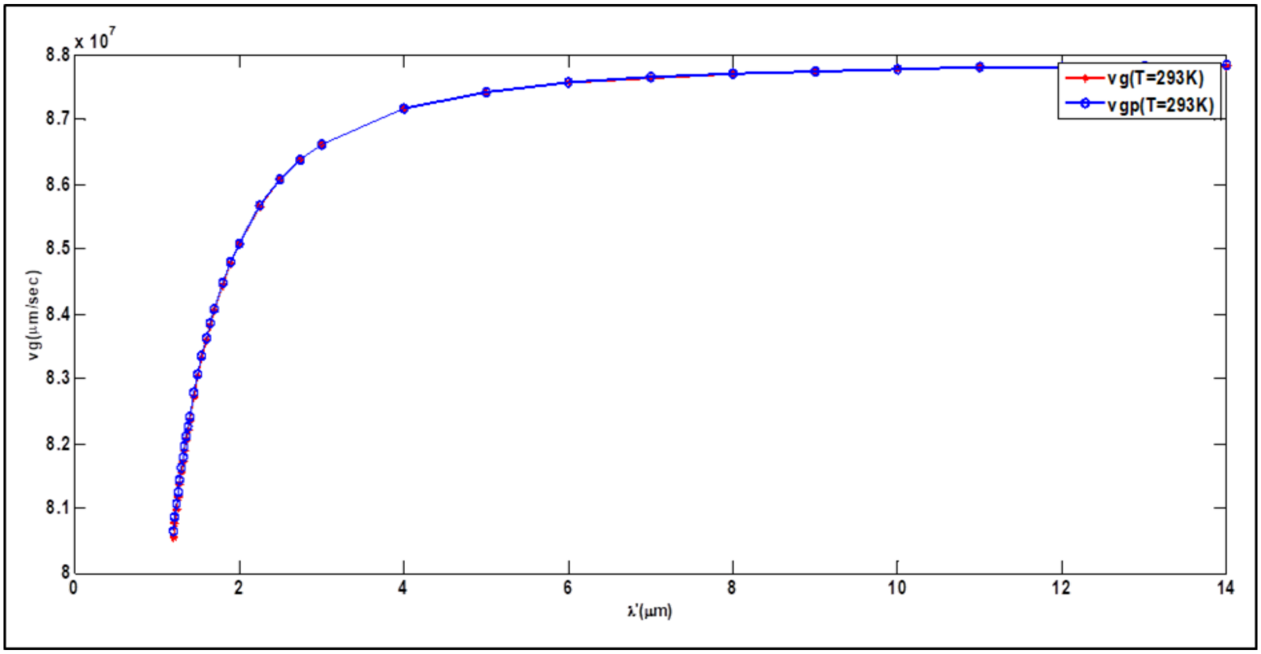
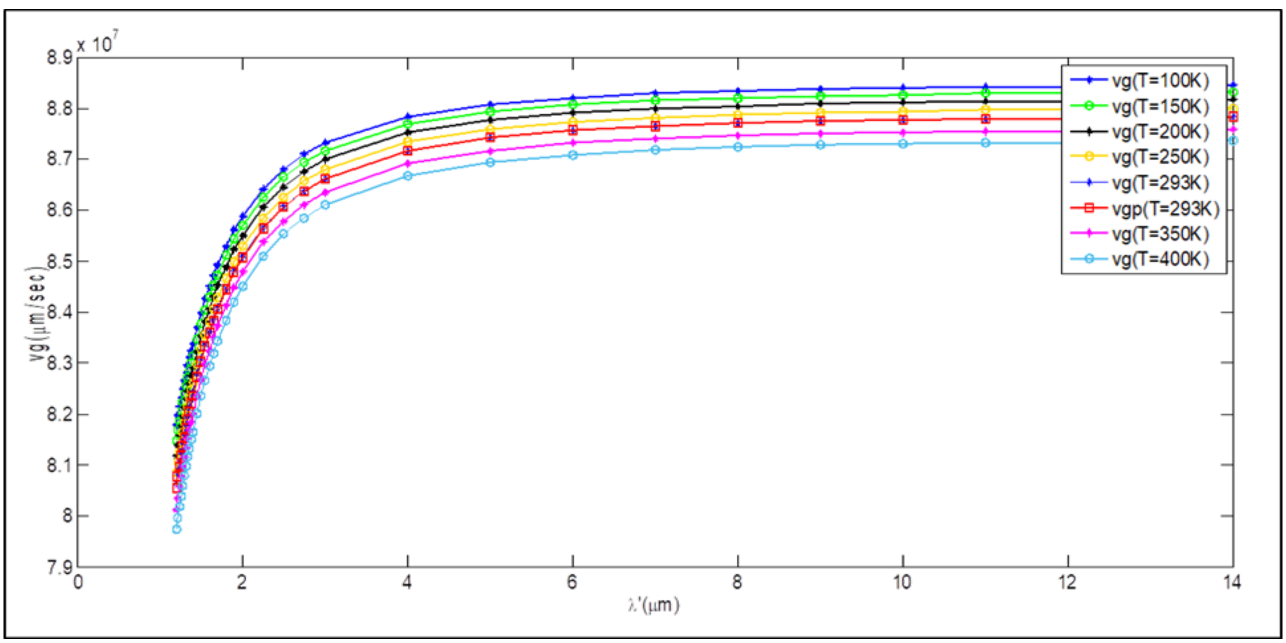


Fig (6): group velocity values calculated using the actual values from equation 5 (vgp) and measured from equation (5). For a 293K temperature.



Fig(7): Group velocity values were calculated using equation (5) at (100-400K).

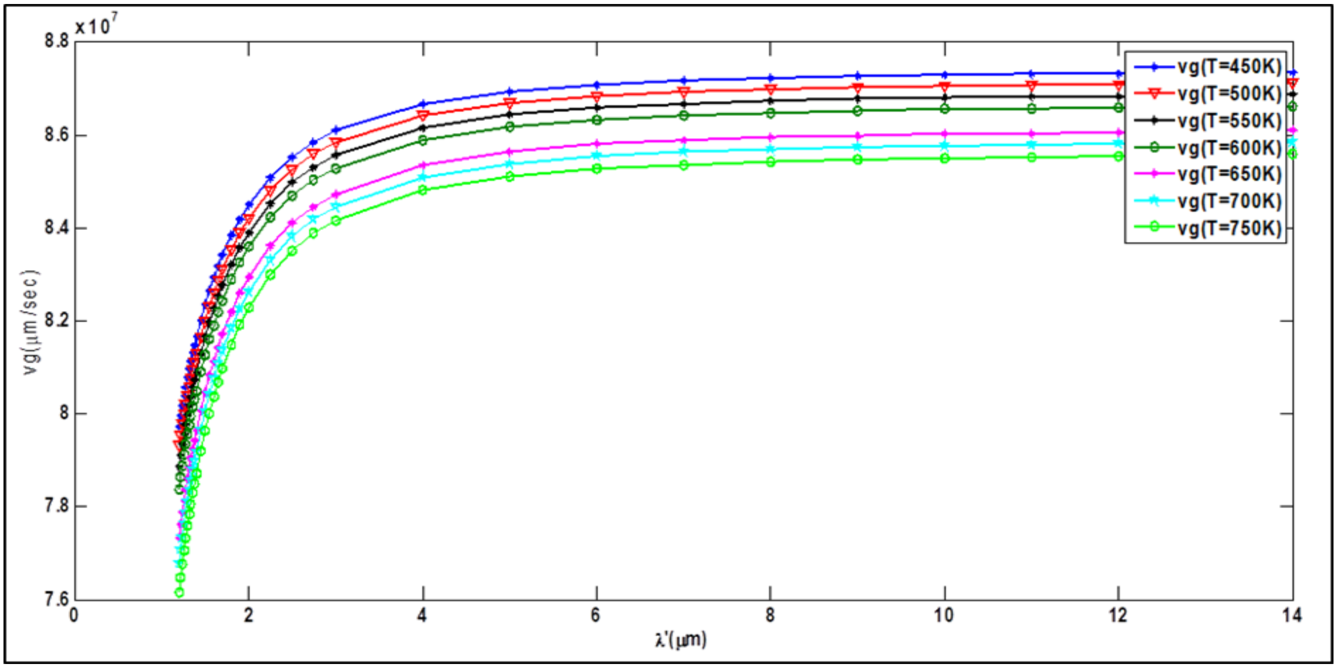
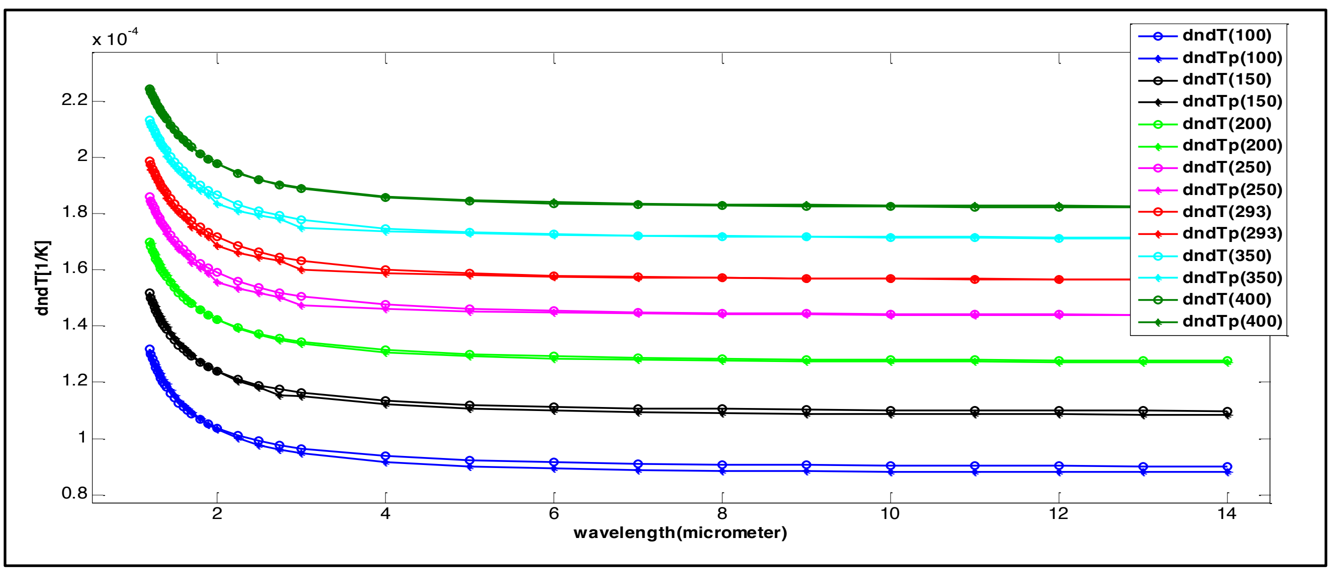
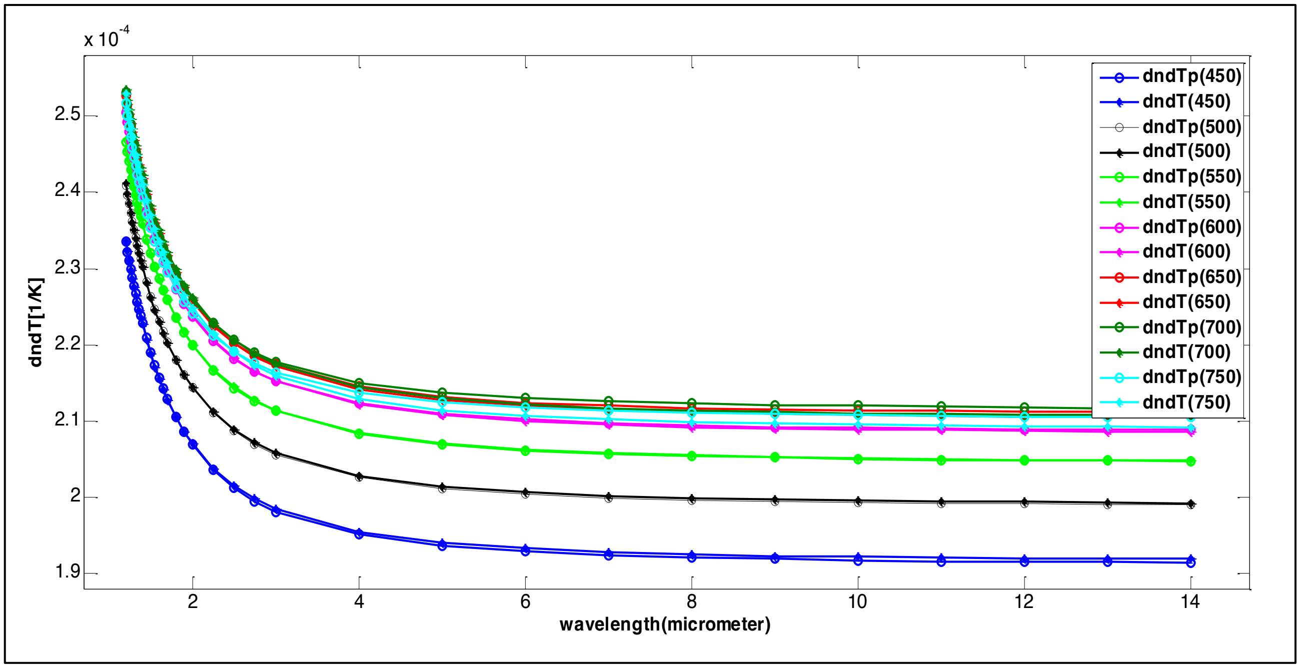


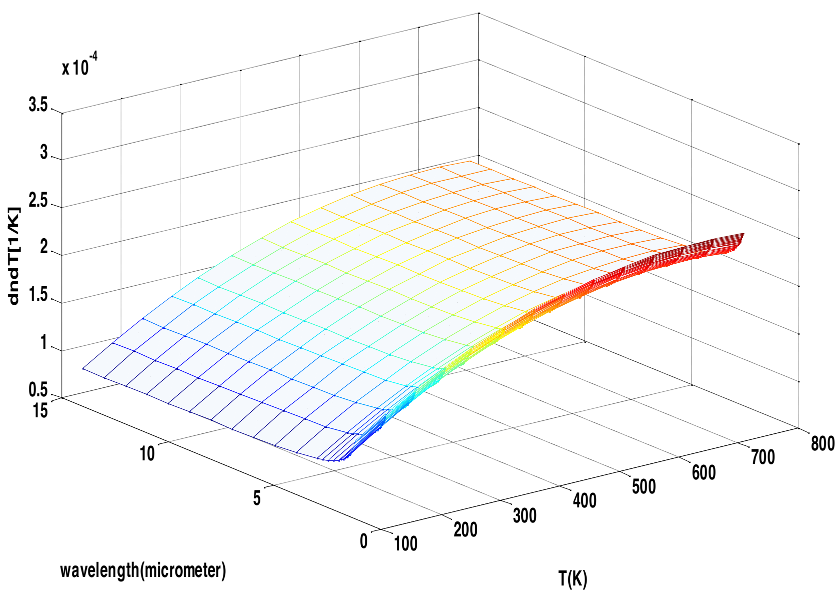
Fig (8):Group velocity values were calculated using equation (5) at (500-750K).



Fig(9) : The thermo-optic coefficient values (dn/dT) were calculated using values from Eq.(6) and reference ref.[4], dn/dTp. For temperatures (100-400)K.



Fig(10): Values of thermo-optic coefficient(dn/dT) cacluated from Eq.(6), dn/dT, with the values from ref.[4], dn/dTp. For temperatures(450-750)K.



Fig(8): The 3-D shows the dependence dn/dT on temperature and wavelength .

5. Conclusions

The optical properties of silicon material were studied, including spectral dispersion, thermo-optic coefficient, and group velocity. Several conclusions in this paper can be summarized as follows:

The spectral dispersion values for silicon electromagnetic wave material were negative, indicating normal dispersion. Spectral dispersion is inversely proportional to temperature and directly proportional to wavelength. The derived dispersion equation (Eq. 3), is suitable, as the difference ratio from Li values is less than 0.002%. The derived equation (Eq. 6), which is based on wavelength and temperature, is suitable for finding thermo-optic coefficient at any temperature and wavelength , is appropriate for temperatures. The thermo-optic coefficient dn/dT is inversely proportional to wavelength and directly proportional to temperature. The group velocity increases rapidly for wavelengths between 1.2 μm and 2.5 μm, then increases very slowly until it approaches a constant value, which depends on the temperature. The derived equation for phase velocity shows that the phase velocity is directly proportional to wavelength and inversely proportional to temperature. It is clear that the phase velocity is greater than the group velocity.

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